PRACTICE PROBLEMS FOR MAT 266 TEST 2

1. Do review problems 41, 43, 44, 46 on page 355 in the text book

2. Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as divergent and explain why.

   (a) $\int_{-5}^{2} \frac{dx}{x^5}$

   (b) $\int_{-\infty}^{\infty} x^2 e^{-x^3}\, dx$

   (c) $\int_{2}^{\infty} \frac{3}{(x+4)^{\frac{3}{2}}}\, dx$

   (d) $\int_{-\infty}^{-3} \frac{dx}{\sqrt{8-x}}$

3. Do review problems 2,3,4 on page 408 in the text book

4. Find the area of the region bounded by the functions $y = 9 - x$, $y = 8x$ and $y = \frac{3x}{2}$ by sketching the region, labeling the intersection points, clearly showing the integral(s) as well as the antiderivative.

5. Do review problems 11, 12, 13, 14 on page 408 in the text book

6. Rotate the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ around the given line. Set up the integral to find the volume of the solid of revolution with the specified method. Include limits of integration. Include a sketch. Do not evaluate.

   (a) about the x-axis using the disk/washer method.

   (b) about the $y = -2$ line using the disk/washer method.

   (c) about the $y = 6$ line using the disk/washer method.

   (d) about the y-axis using the disk/washer method.

   (e) about the $x = -3$ line using the disk/washer method.

   (f) about the $x = 8$ line using the disk/washer method.

   (g) about the x-axis using the shell method.

   (h) about the $y = -2$ line using the shell method.

   (i) about the $y = 6$ line using the shell method.

(j) about the y-axis using the shell method.

(k) about the $x = -3$ line using the shell method.

(l) about the $x = 8$ line using the shell method.

7. Find the length of the arc for the given functions.Clearly show the integral(s) as well as the antiderivative.

   (a) $f(x) = \dfrac{x^3}{3} + \dfrac{1}{4x}$ from $x = 1$ to $x = 3$.

   (b) $f(x) = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ from $x = 0$ to $x = 3$.

8. Do review problem 29 on page 409 in the text book

9. If the work required to stretch a spring 0.4 m beyond its natural length is 6 J, how much work is required to stretch it 0.2 m beyond its natural length? Clearly show the integral(s) as well as the antiderivative.

10. An aquarium is in the given shape. It is full of water. Set up the integral to find the work that is required to pump all of the water over the top. The mass density of the water is 1000 $\dfrac{kg}{m^3}$

    (a) 2 m long, 1 m wide and 1 m deep rectangular box.

    (b) 2 m high, 1 m radius cylinder.

    (c) 2 m high, 1 m radius inverted cone.

11. Repeat problem 10 with the aquarium half full.

SOLUTIONS

1. See odd number solutions in the book. Ex44.: $\frac{40}{3}$ Ex46.: Diverges

2. (a) Divergent (b) Divergent (c) $\sqrt{6}$ (d) Divergent

3. Ex2.: $\frac{512}{3}$ Ex4.: $\frac{32}{3}$

4. 11.7

5. Ex12.: $\int_0^2 2\pi(8 - x^3)(2 - x)\,dx$ Ex14.: (a) $\frac{5}{12}$ (b) $\frac{41\pi}{105}$ (c) $\frac{13\pi}{30}$

6. (a) $\displaystyle\int_0^1 \pi(e^x)^2\,dx$

   (b) $\displaystyle\int_0^1 \pi((e^x + 2)^2 - 2^2)\,dx$

   (c) $\displaystyle\int_0^1 \pi(6^2 - (6 - e^x)^2)\,dx$

(d) $\displaystyle\int_0^1 \pi 1^2 \, dy + \int_1^2 \pi(1^2 - (\ln y)^2) \, dy$

(e) $\displaystyle\int_0^1 \pi(4^2 - 3^2) \, dy + \int_1^e \pi(4^2 - (\ln y + 3)^2) \, dy$

(f) $\displaystyle\int_0^1 \pi(8^2 - 7^2) \, dy + \int_1^e \pi((8 - \ln y)^2 - 7^2) \, dy$

(g) $\displaystyle\int_0^1 2\pi y 1 \, dy + \int_1^e 2\pi y(1 - \ln y) \, dy$

(h) $\displaystyle\int_0^1 2\pi(y+2) 1 \, dy + \int_1^e 2\pi(y+2)(1 - \ln y) \, dy$

(i) $\displaystyle\int_0^1 2\pi(6-y) 1 \, dy + \int_1^e 2\pi(6-y)(1 - \ln y) \, dy$

(j) $\displaystyle\int_0^1 2\pi x e^x \, dx$

(k) $\displaystyle\int_0^1 2\pi(x+3) e^x \, dx$

(l) $\displaystyle\int_0^1 2\pi(8-x) e^x \, dx$

7. (a) $\frac{53}{6}$ (b) 12

8. see book

9. 1.5 J

10. (a) $\displaystyle\int_0^1 1000 \cdot 9.8 \cdot 2 \cdot y \, dy$ ($y = 0$ on the top)

(b) $\displaystyle\int_0^2 1000 \cdot 9.8 \cdot \pi \cdot y \, dy$ ($y = 0$ on the top)

(c) $\displaystyle\int_0^2 1000 \cdot 9.8 \cdot \pi(\frac{y}{2})^2(2-y) \, dy$ ($y = 0$ on the bottom)

11. (a) $\displaystyle\int_{0.5}^1 1000 \cdot 9.8 \cdot 2 \cdot y \, dy$

(b) $\displaystyle\int_1^2 1000 \cdot 9.8 \cdot \pi \cdot y \, dy$

(c) $\displaystyle\int_0^1 1000 \cdot 9.8 \cdot \pi(\frac{y}{2})^2(2-y) \, dy$