MAT 092 · Lecture 1

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Outline

Today's lesson will cover some basic concepts of arithmetic. Toward the end of the lesson, we will solve a few problems using these concepts. The chapters that will be covered day are:

- §1.3 and 1.5–Elementary Arithmetic
- §1.8, 1.9, and 1.10–Applications

§1.3 Arithmetic

The commutative law says that when you add or multiply numbers, the order you add or multiply them does not matter. In other words,

$$a + b = b + a.$$

The distributive law says that when a number is multiplied by the sum of two numbers that it equals, that number multiplied by the two numbers individually, with the results added afterward; that is,

$$(a + b) \cdot c = a \cdot c + b \cdot c.$$

§1.3 Order of Operations

When a number is both preceded and followed by two binary operations that are not the same, a rule is required for which operation should be applied first; this rule is known as the order of operations. So binary operations within parentheses outrank exponents, exponents outrank multiplication and division (but multiplication and division have equal rank), and these two outrank addition and subtraction (which have equal rank).

§1.5 Adding and Subtracting Fractions

When adding fractions with the same denominator; the result is the same as the sum of the numerators over the denominator. A similar statement can be made for subtraction. If the fractions have different denominators,

- First, find the least common denominator.
- Then write equivalent fractions using this denominator.
- Add or subtract the fractions.

§1.5 Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers, simply convert the mixed numbers into improper fractions, then add or subtract them as fractions.

Example :

$$9\frac{1}{2} + 5\frac{3}{4} = \ ?$$

§1. 5 Multiplying Fractions and Fractions

When two fractions are multiplied, the result is a fraction with a numerator that is the product of the fractions' numerators and a denominator that is the product of the fractions' denominators.

Example :

$$\frac{4}{7} \times \frac{5}{11} = \ ?$$

§1.5 Multiplying Mixed Numbers

To multiply mixed numbers, convert them to improper fractions and multiply.

Example:

$$4\frac{1}{5} \times 2\frac{2}{3} = \ ?$$

§1.5 Reciprocal

The reciprocal of a fraction is obtained by switching its numerator and denominator. To find the reciprocal of a mixed number, first convert the mixed number to an improper fraction, then switch the numerator and denominator of the improper fraction. Notice that when you multiply a fraction and its reciprocal, the product is always 1.

Example:
Find the reciprocal of $\frac{31}{75}$. We switch the numerator and denominator to find the reciprocal: $\frac{75}{31}$.

§1.5 Dividing Fractions

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Example:

$$7 \div \frac{1}{5} = 7 \times \frac{5}{1} = 7 \times 5 = 35.$$

§1.5 Dividing Mixed Numbers

To divide mixed numbers, you should always convert to improper fractions, then multiply the first number by the reciprocal of the second.

Example:

$$1\frac{1}{2} \div 3\frac{1}{8} = \ ?$$

§1.8 Percentage

To convert a fraction to a percentage, divide the numerator by the denominator. Then move the decimal point two places to the right (which is the same as multiplying by 100) and add a percent sign.

Example: Given the fraction $\frac{5}{8}$ what is the percentage?

To change a percentage to a fraction, divide it by 100 and reduce the fraction or move the decimal point to the right until you have only integers.

§1.9 Proportions

A proportion is a name we give to a statement that two ratios are equal. It can be written in two ways:

$$\frac{a}{b} = \frac{c}{d}$$

or

$$a : b = c : d.$$

§1.13 Calculations: converting from one unit to another

The simplest way to carry out calculations involving different units is to use the dimensional method. In this method, a quantity described in one unit is converted to another by means of a conversion factor. A conversion factor is ratio of two quantities that have different units:

$$\text{start quantity} \times \text{conversion factor} = \text{equivalent quantity}.$$

§1.15 Integers

The set containing the natural numbers, zero, and all negative numbers is called the integers. The symbol $\mathbb{Z}$ is used to denote set of integers. Some important properties of the integers:

- The sum or product of two integers is always an integer.
- The integers is an ordered set.

§1.5 Addition and Subtraction of Integers

- Subtracting of an integer is equivalent to adding the opposite or inverse of that integer.
- The absolute value of an integer represents the distance of that integer from zero.
- When adding two integers of the same, add the absolute value of the integers. The sign of the sum is the same as sign of both integers.
- When adding two integers of opposite signs, determine their absolute value and then subtract the smaller number from larger number. The sign of the difference is the sign of larger number.

§1.16 Multiplication and Division of Integers

- The product or quotient of two integers of the same sign is always positive.
- The product or quotient to two integers of opposite sign is always negative.
- The quotient of two integers is not always an integer.

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§4.1 Inequality symbols

- $\neq$ means not equal to
- $<$ means is less than
- $>$ means is greater than
- $\leq$ means less than or equal to
- $\geq$ means greater than or equal to

§4.2 Solving inequalities

For real numbers a, b, and c.

- If $a < b$, then $a + c < b + c$.
- If $a < b$, then $a + c < b - c$.

Similar statements can be made for the symbols $>, \leq$, and $\geq$.

For real numbers a, b, and c.

- If $a < b$ and $c > 0$, then $ac < bc$.
- If $a < b$ and $c < 0$, then $ac > bc$.
- If $a < b$ and $c > 0$, then $a/c < b/c$.
- If $a < b$ and $c < 0$, then $a/c > b/c$.

Similar statements can be made for the symbols $>, \leq$, and $\geq$.

§4.3 Exponents and Polynomials

- If n is a natural number, then $x^n = \overbrace{x \cdot x \cdots x}^{n \text{ factors of } x}$.
- If n and m are natural numbers, then $x^m \cdot x^n = x^{m+n}$.
- If n and m are natural numbers, then $(x^m)^n = x^{m \cdot n}$.
- If n is a natural number, then $(xy)^n = x^n y^n$ and if $y \neq 0$, then $(x/y)^n = x^n / y^n$.

§4.4 The FOIL method

FOIL is an acronym for First terms, Outer terms, Inner terms, and Last terms. To use the FOIL method to multiply $2x - 4$ by $3x + 5$, we

- Multiply the First terms $2x$ and $3x$ to get $6x^2$,
- Multiply the Outer terms $2x$ and 5 to get $10x$,
- Multiply the Inner terms -4 and $3x$ to get $-12x$, and
- Multiply the Last terms -4 and $+5$ to get -20.

$$\begin{aligned}(2x-4)(3x+5) &= 6x^2 + 10x + -12x - 20\\ &= 6x^2 - 2x - 20\end{aligned}$$

§4.5 Factoring Trinomials of the form $x^2 + bx + c$

- Find two integers whose product is constant term in the trinomial.
- There sum must equal the coefficient of the middle term.

§4.6 Factoring Trinomials of the form $ax^2 + bx + c$

- Find the factors of the leading coefficient.
- Find the factors of the last term.
- Try combination of the first terms and last terms until you find one that works.

§3.4 Linear functions

The slope of a line function is defined by

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1}, \end{aligned}$$

Where $x_1 \neq x_2$.

§3.4 Geometric Meaning of Slope

From its geometric meaning, slope may be determined from a graph by the formula

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{distance up or down}}{\text{distance right or left}} \end{aligned}$$

§3.5 Horizontal and Vertical Intercepts

Definition: A horizontal intercept of a graph is a point where the graph crosses the horizontal axis. The ordered pair notation for a horizontal intercept has the form $(a, 0)$, where a is the input value.

Definition: A vertical intercept of a graph is a point where the graph crosses the vertical axis. The ordered pair notation for a vertical intercept has the form $(0, b)$, where b is the output value.