MAT209 EXAM I

MAT 209 · SPRING 2009

You must show all work to get full credit.

Problem 1 (20 points)**.** Solve the differential equation

$$\frac{dy}{dx} = y^2$$

subject to the initial condition $y(0) = 1$.

Problem 2 (20 points)**.** Solve the differential equation

$$\frac{dy}{dx} = -xy$$

subject to the initial condition $y(0) = \frac{1}{\sqrt{2\pi}}$.

Problem 3 (10 points)**.** Consider the initial value problem

$$\frac{dy}{dx} = x^2 - y$$

where $y(0) = 1$. Can this ODE be solved using *Separation of Variables*? If not, What other methods that we have learned could be used to solve initial value problem?

Problem 4 (20 points)**.** Consider the initial value problem

$$\frac{dy}{dx} = x - y$$

where $y(0) = 1$. Use the Eulers method with step size 0.20 to find the approximate value of $y(1)$.

Problem 5 (40 points)**.** Use geometric analysis to analyze the differential equation

$$\frac{dy}{dt} = (y-1)(y-3)$$

Your answer should include:

a) a graph of $g(y)$ vs. y , where $g(y)$ is the derivative of y as a function of y.
b) sketches of the solution curves $y(t)$ for the initial value problem $y(0) = 2$ and all steady state solutions.
c) show concavity and show the y of all inflection points for solution curves.
d) stability analysis for all steady state solutions.

Problem 6. *Extra Credit* [5 points] Consider the ODE

$$\frac{dy}{dx} = y - 1$$

Verify that for any constant C the expression $y = Ce^x + 1$ is a solution to the ODE.

Date: March 16, 2009 *Due Date*: Wednesday, March 18, 2009.

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