

EXAM II

MAT 209 · SPRING 2009

You must show all work to get full credit.

Problem 1 (25 points). An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\begin{aligned}\frac{dN_1}{dt} &= 0.1N_1 \left(1 - \frac{N_1}{100} - \frac{N_2}{50} \right) \\ \frac{dN_2}{dt} &= 0.3N_2 \left(1 - \frac{N_1}{100} - \frac{N_2}{100} \right)\end{aligned}$$

- (a) Determine the steady state solution of this system.
(b) Based on the above model, would you characterize the species as competitive? In the long term to which of the possible steady state solutions will the populations tend? Explain

Problem 2 (25 points). A lake holds a population of 200,000 salmon. The natural growth rate of the salmon population is 5%. Fishing removes 30,000 salmon each year from the lake. Set up a differential equation modeling the fish population as a function of time. Solve it to find out if and when the fish population will be completely depleted.

Problem 3 (25 points). 100 batteries were randomly selected from a large batch and placed through a simulation of everyday use until they burned out. The table below summarizes the distribution of their lifetime (=number of hours until burn out). For example, 20 batteries lasted between a little more than 10 hours and up to 15 hours.

TABLE 1

Hours until burnout	(5, 10]	(10, 15]	(15, 20]	(20, 25]	(25, 30]
# of batteries	5	20	40	25	10

- a) Based on the data, prepare a relative frequency histogram for the time until burnout for battery from the sample.
b) Based on data, estimate as accurately as you can the median number of hours that a sample battery lasted until burnout.
c) Based on the data estimate as accurately as you can the average number of hours that a sample battery lasted until burnout.

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Problem 4 (20 points).

- (a) Consider the four points $(8, 17)$, $(2, 5)$, $(6, 13)$, and $(4, 9)$. Find the quantities s_x , s_y , c_{xy} , and r and use these to find the regression line.
- (b) [5 points] What estimate would you give for the value of y when $x = 3$?

Problem 5. (Extra Credit)[10 points] You purchase a new car and keep it for 10 years. Every year you record the amount you spend on repairs, including routine maintenance. Which of the following would you expect for the correlation coefficient?

- (a) moderately positive
- (b) close to zero
- (c) moderately negative
- (d) zero