HOMEWORK SET 4 SOLUTION

MAT209 · SPRING 2009

You must show all work to get full credit. You can use a calculator to check your work.

Problem 1. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= x - 3y \\ \frac{dy}{dt} &= x + y - 2. \end{aligned}$$

Find all the steady state solutions of this system. Draw the null-clines and all steady state solutions. Analyze the stability of the steady state solutions.

Solution: First, begin the problem by finding the steady state solution to the system of differential equations. That is, set $x' = y' = 0$. Thus,

$$\begin{aligned} 0 &= x - 3y \\ 0 &= x + y - 2 \end{aligned}$$

It follows from the first equation that $y = \frac{1}{3}x$. Substitute this result for y into the second equation to get

$$\begin{aligned} 0 &= x + \frac{1}{3}x - 2 \\ &= \frac{4}{3}x - 2. \end{aligned}$$

But then, $x = \frac{3}{2}$. Now, $y = \frac{1}{2}$. Therefore, $(\frac{3}{2}, \frac{1}{2})$ is steady state solution.

We turn our attention to finding the null-clines. $y = \frac{1}{3}x$ is a null-cline. The other other null-cline is determined from second equation $0 = x + y - 2$. We rewrite the equation as a function of x; that is, $y = 2 - x$. The two null-clines divide first quadrant of Cartesian coordinate system into four regions as shown in the figure on the next page:

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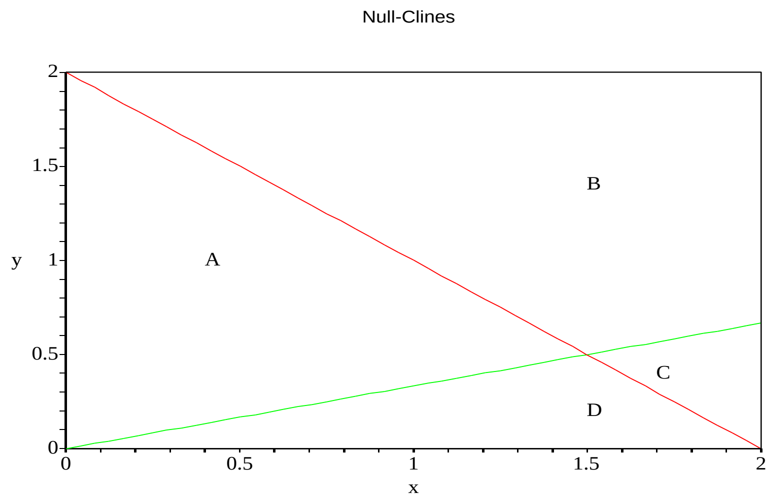

Fig. 1 y *vs* x

The point $(1.4, 0.5)$ is in region A. The value of both x' and y' at this point is -0.1. The curve $(x(t), y(t))$ will move 45 degree in a southwesterly direction away from the steady state. Now, $(1.5, 0.4)$ is in region D. The value of x' and y' are $+0.3$ and -0.1, respectively. The curve $(x(t), y(t))$ will move 71.6 degree in a southeasterly direction away from the steady state. $(1.6, 0.5)$ is in region C, and the value of both x' and y' is $+0.1$. $(x(t), y(t))$ will move 45 degree in a northeast direction away from the steady state. The $(1.5, 0.6)$ point is in region B, the value of x' and y' are -0.3 and 0.1 respectively. $(x(t), y(t))$ will move way from the steady state at an angle of 71.6 degrees in a northwesterly direction. Consequently, $(1.5, 0.5)$ is not a stable steady state.

Problem 2. Suppose you attain an average of 73 on three exams during the first half of a course. Five exams are scheduled for the second half. What must your average be on the remaining five exams to finish with an average of 80 for the entire eight exams?

Solution: Let $X_1, X_2, ..., X_8$ denote your exam scores for the eight exams. For the first half of the course you attain an average of 73; that is,

$$73 = \frac{X_1 + X_2 + X_3}{3}.$$

But then, $X_1 + X_2 + X_3 = 219$. You want an average of 80 for the entire course. So,

$$80 = \frac{X_1 + X_2 + \cdots + X_8}{8}.$$

This implies that $X_1 + X_2 + \cdots + X_8 = 640$. But then, $X_4 + X_5 + \cdots + X_8 = 421$.

It follows that

$$\frac{X_4 + X_5 + \cdots + X_8}{5} = \frac{421}{5} = 84.2.$$

Problem 3. Suppose a data collection S consist of the six numbers $7, 3, 5, 2, 1, 2$.

(a) Find the data value when adjoined to S will give a data set S^* having a mean of 4.

(b) Find the median of the original collection S.

(c) Suppose any number whatsoever (not necessarily a whole number) is adjoined to the collection S, produces a collection S'. Explain why the median for S' differs from the median of S by at most $\frac{1}{2}$.

Solution: (a) Let x denote the unknown data value. Hence, $S^* = \{7, 3, 5, 2, 1, 2, x\}$. It follows that

$$\begin{aligned} 4 &= \frac{1+2+2+3+5+7+x}{7} \\ &= \frac{20+x}{7}. \end{aligned}$$

But then, $20 + x = 28$. So $x = 8$. (b) Let S'' denote the ordered collection S; that is, $S'' = \{1, 2, 2, 3, 5, 7\}$. Therefore, the median is

$$\frac{1}{2}(2+3) = \frac{5}{2} = 2.5.$$

(c) Let x denote the unknown data value. So, $S' = \{7, 3, 5, 2, 1, 2, x\}$.

Case 1: If $x \geq 3$, the median is 3. So there is a difference of $\frac{1}{2}$ between the median of S and S'.

Case 2: If $x \leq 2$, the median is 2. Again, there is a difference of $\frac{1}{2}$ between the median of S and S'.

Case 3: If $2 < x < 3$, the median is x. The difference between the median S and S', $|x - \frac{5}{2}| < \frac{1}{2}$.

Therefore the median for S' differs from the median of S by at most $\frac{1}{2}$.

Problem 4. 50 students take a multiple-choice exam with 10 questions. The number of correct and incorrect answers is recorded for each student. Which of the following values would you consider most likely for the correlation coefficient? Justify your choice.

(a) +1

(b) 0.5

(c) 0

(d) -0.5

(e) -1

Solution: Consider an experiment with a finite number of outcomes. The outcomes are represented by E. E can be $e_1, e_2, ..., e_n$, where e_i is the ith outcome for $1 \leq i \leq n$. Each outcome has a weight (probability) associated with it. So E is called a random variable.

Let X and Y represent the number of correct and wrong answers a student had on the multiple-choice exam, respectively. Note that the value of both X and Y is subject to chance for each student; so X and Y are called random variables. Also notice that sum of

X and Y is 50. So X and Y are dependent variables; in other words, if you know X, then you also know Y. We can write Y as follows:

$$Y = 50 - X.$$

It follows that Y decreases linear with X. So the data shows a strong negative linear correlation. By the correlation property, r must be close to -1. The appropriate answer is (e).