PRACTICE EXAM II SOLUTIONS

MAT 209 · SPRING 2009

You must show all work to get full credit.

Problem 1 (10 points). An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\begin{aligned} \frac{dN_1}{dt} &= -0.1N_1(10 - N_2) \\ \frac{dN_2}{dt} &= 0.2N_2(20 - N_1) \end{aligned}$$

(a) Determine the steady state solution of this system.
(b) Explain (in one brief sentence) the significance of the term $-N_2$ in the first equation, and the term $-N_1$ in the second, for the competitive process.

Solution: (a) To determine the steady state solutions of this system, set $N_1' = N_2' = 0$. That is,

$$\begin{aligned} -0.1N_1(10 - N_2) &= 0 \\ 0.2N_2(20 - N_1) &= 0. \end{aligned}$$

It follows that $N_1 = 0$ and $N_2 = 10$ are solutions to equation one. Now, both solutions have to also satisfy the second equation as well. Substituting zero for N_1 in equation two, we see that $N_2 = 0$. So, $(0, 0)$ is a steady state. Now, substitute 10 for N_2 in equation two. This yields $N_1 = 20$. And so, $(20, 10)$ is also a steady state.

(b) The term $-N_2$ in the first equation is the interspecies impact that species two (N_2) has on species one (N_1), and $-N_1$ in the second is the interspecies impact that species one has on species two.

Problem 2 (10 points).

(a) For the data in the table below, the regression line has equation $y = 0.375 + 0.25x$ (or, $y = \frac{3}{8} + \frac{x}{4}$).

 (i) Use least squares method to find how well the line fits the data.
 (ii) What would be the predicted y for $x = \frac{1}{2}$?

x	0	1	1	2
y	0.5	0	1	1

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(b) Suppose a set of bivariate data is plotted on semilog paper ($z = log(y)$ vs. x), and a linear relation appears, with regression line $z = 2 + .5x$. What equation would we expect for y vs. x?

Solution: (a) Using the *Least Squares Method*, we find that

$$\begin{aligned} f(0.375, 0.25) &= \sum_{i=1}^{4}(0.375 - 0.25x_i - y_i)^2 \\ &= (0.375 + 0.25 \cdot 0 - 0.5)^2 + (0.375 + 0.25 \cdot 1 - 0)^2 + (0.375 - 0.25 \cdot 1 - 1)^2 \\ &\quad + (0.375 + 0.25 \cdot 2 - 1)^2 \\ &= 0.5625. \end{aligned}$$

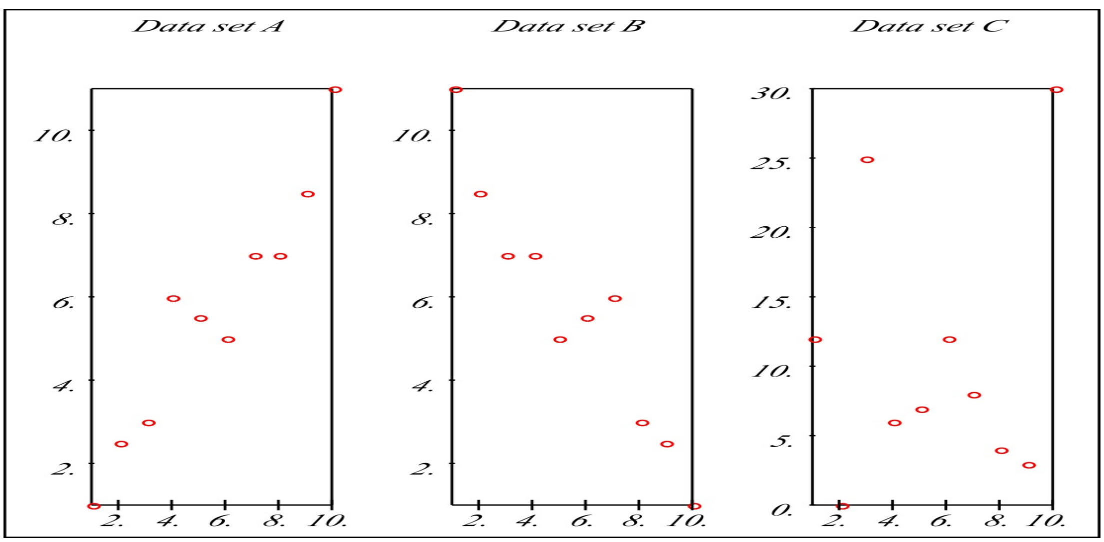
So the value of f is very close to zero. This would suggest that the regression line is a good fit. The predict value of y at $x = 0.5$ is 0.5.

(b) Now,

$$\begin{aligned} y &= 10^{log(y)} \\ &= 10^z \\ &= 10^{2+0.5x} \\ &= 10^2 \cdot 10^{0.5x} \\ &= 100 \cdot 10^{0.5x}. \end{aligned}$$

Problem 3 (10 points)**.**

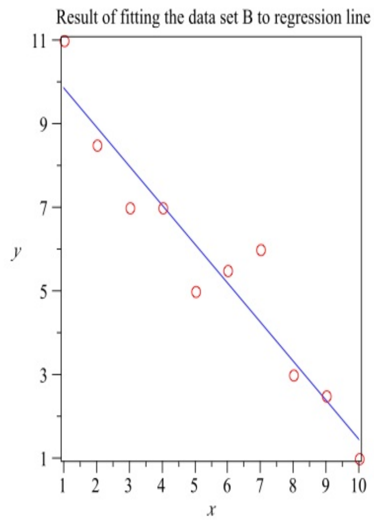
(a) Put the three graphs below in order of increasing correlation coefficient r. Smallest r to largest r



(b) Sketch your best guess as to the regression line of the data set B, and write down the equation of that line.

Solution: (a) data set B's r value is close to negative one, data set C's r value is close to zero, and data set A's r value is close to one.

(b)



From the plot, we can see by inspection that the regression line passes through the points (1,10) and (10,1.5). It follows that

$$\begin{aligned} \hat{b} &= \frac{1.5 - 10}{10 - 1} = -0.94 \\ y - 10 &= -0.94(x - 1) \end{aligned}$$

But then, the equation of regression line is $y = 10.94 - 0.94x$.

Problem 4 (8 points).

(a) Consider the four points $(1, 2), (5, 1), (2, 2)$, and $(0, 8)$. Find the quantities, and r and use these to find the regression line. s_x, s_y

(b) [2 points] What estimate would you give for the value of y when $x = 4$?

Solution: Recall that

$$\begin{aligned} \hat{b} &= \frac{c_{xy}}{s_x^2} \text{ is the slope of the regression line.} \\ \hat{a} &= \overline{y} - \hat{b}\overline{x} \text{ is the } y\text{-intercept of the regression line.} \end{aligned}$$

Now, we must determine $\overline{x}, \overline{y}, c_{xy}, r, s_y$, and s_x. So,

$$\begin{aligned}
\overline{x} &= \frac{0+1+2+5}{4} = 2 \\
\overline{y} &= \frac{8+2+2+1}{4} = 3.25 \\
c_{xy} &= \frac{(0-2)(8-3.25)+(1-2)(2-3.25)+(2-2)(2-3.25)+(5-2)(1-3.25)}{3} = -5 \\
s_x &= \sqrt{\frac{(0-2)^2+(1-2)^2+(2-2)^2+(5-2)^2}{3}} = 2.16 \\
s_y &= \sqrt{\frac{(8-3.25)^2+(2-3.25)^2+(2-3.25)^2+(1-3.25)^2}{3}} = 3.20 \\
r &= \frac{-5}{(2.16)(3.20)} = -0.723.
\end{aligned}$$

So $\hat{b} = -1.071$. Now,

$$\hat{a} = 3.25 - (-1.071)(2) = 5.392.$$

Therefore the equation for the regression line is $y = 5.392 - 1.071x$. (b) The predicted value of y at $x = 4$ is 1.107.