HOMEWORK SET 2

MAT 217 · FALL 2008

You must show all work to get full credit. You can use a calculator to check your work.

Problem 1. Find the solution to the system of equations below using the Gauss-Jordan Method

$$\begin{cases} 2x_1 + x_2 & = & 8 \\ x_1 + 3x_2 & = & 9 \end{cases}$$

Solution: Find the augmented matrix for the system of equations. Thus,

$$(A|b) = \left[\begin{array}{cc|c} 2 & 1 & 8 \\ 1 & 3 & 9 \end{array}\right]$$

Using the Gauss-Jordan Method, we get

$$\left[\begin{array}{cc|c} 2 & 1 & 8 \\ 1 & 3 & 9 \end{array}\right] \xrightarrow{0.5R1} \left[\begin{array}{cc|c} 1 & 0.5 & 4 \\ 1 & 3 & 9 \end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{cc|c} 1 & 0.5 & 4 \\ 0 & 2.5 & 5 \end{array}\right] \xrightarrow{0.4R2} \left[\begin{array}{cc|c} 1 & 0.5 & 4 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R1-0.5R2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

It follows that $x_1 = 3$ and $x_2 = 2$.

Problem 2. Use Gauss-Jordan Method to solve the linear system

$$\begin{cases} 2x_1 + 4x_2 - 2x_3 & = & 2 \\ 4x_1 + 9x_2 - 3x_3 & = & 8 \\ -2x_1 - 3x_2 + 7x_3 & = & 10 \end{cases}$$

Solution: Find the augmented matrix for the system of equations. Thus,

$$(A|b) = \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 9 \end{array}\right]$$

Using the Gauss-Jordan Method, we get

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 9 \end{array}\right] \xrightarrow{0.5R1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 9 \end{array}\right] \xrightarrow{R2-4R1,\, R3-2R1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 12 \end{array}\right]$$

$$\xrightarrow{R1-2R2,\, R3-R2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -7 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 4 & 8 \end{array}\right] \xrightarrow{0.25R3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -7 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R1+3R3,\, R2-R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

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It follows that $x_1 = -1, x_2 = 2$, and $x_3 = 2$.

Problem 3. Find the solution to the system of equations below using the Gauss-Jordan Method

$$\begin{cases} 3x_1 + 7x_2 + 4x_3 + 4x_4 &= -7 \\ 7x_1 + 7x_2 + 4x_3 + 7x_4 &= 3 \\ 4x_1 + 3x_2 + 2x_3 + 3x_4 &= 2 \\ 3x_1 + 2x_2 + x_3 + 3x_4 &= 4 \end{cases}$$

Solution: Find the augmented matrix for the system of equations. Thus,

$$(A|b) = \left[\begin{array}{cccc|c} 3 & 7 & 4 & 4 & -7 \\ 7 & 7 & 4 & 7 & 3 \\ 4 & 3 & 2 & 3 & 2 \\ 3 & 2 & 1 & 3 & 4 \end{array}\right]$$

Using the Gauss-Jordan Method, we get

$$\left[\begin{array}{cccc|c} 3 & 7 & 4 & 4 & -7 \\ 7 & 7 & 4 & 7 & 3 \\ 4 & 3 & 2 & 3 & 2 \\ 3 & 2 & 1 & 3 & 4 \end{array}\right] \xrightarrow{1/3R1} \left[\begin{array}{cccc|c} 1 & \frac{7}{3} & \frac{4}{3} & \frac{4}{3} & -\frac{7}{3} \\ 7 & 7 & 4 & 7 & 3 \\ 4 & 3 & 2 & 3 & 2 \\ 3 & 2 & 1 & 3 & 4 \end{array}\right] \xrightarrow{R2-7R1, R3-4R1, R4-3R3}$$

$$\left[\begin{array}{cccc|c} 1 & \frac{7}{3} & \frac{4}{3} & \frac{4}{3} & -\frac{7}{3} \\ 0 & -28/3 & -16/3 & -7/3 & 58/3 \\ 0 & -\frac{19}{3} & -\frac{10}{3} & -\frac{7}{3} & \frac{34}{3} \\ 0 & -5 & -3 & -1 & 11 \end{array}\right] \xrightarrow{-\frac{3}{28}R2} \left[\begin{array}{cccc|c} 1 & \frac{7}{3} & \frac{4}{3} & \frac{4}{3} & -\frac{7}{3} \\ 0 & 1 & 4/7 & 1/4 & -29/14 \\ 0 & -\frac{19}{3} & -\frac{10}{3} & -\frac{7}{3} & \frac{34}{3} \\ 0 & -5 & -3 & -1 & 11 \end{array}\right]$$

$$\xrightarrow{R1-7/3R2, R3+19/3R2, R4+5R2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{4} & \frac{5}{2} \\ 0 & 1 & 4/7 & 1/4 & -29/14 \\ 0 & 0 & \frac{2}{7} & -\frac{3}{4} & -\frac{25}{14} \\ 0 & 0 & -\frac{1}{7} & \frac{1}{4} & \frac{9}{14} \end{array}\right] \xrightarrow{7/2R3}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{4} & \frac{5}{2} \\ 0 & 1 & 4/7 & 1/4 & -29/14 \\ 0 & 0 & 1 & -\frac{21}{8} & -\frac{25}{4} \\ 0 & 0 & -\frac{1}{7} & \frac{1}{4} & \frac{9}{14} \end{array}\right] \xrightarrow{R2-4/7R3, R4-1/7R3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{4} & \frac{5}{2} \\ 0 & 1 & 0 & 7/4 & 3/2 \\ 0 & 0 & 1 & -\frac{21}{8} & -\frac{25}{4} \\ 0 & 0 & 0 & -\frac{1}{8} & -\frac{1}{4} \end{array}\right] \xrightarrow{-8R4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{4} & \frac{5}{2} \\ 0 & 1 & 0 & 7/4 & 3/2 \\ 0 & 0 & 1 & -\frac{21}{8} & -\frac{25}{4} \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R1+3/4R4, R2+7/4R4, R3-21/8R4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

It follows that $x_1 = 1, x_2 = -2, x_3 = -1$, and $x_4 = 2$.

Problem 4. In mathematics, an elementary matrix is a simple matrix, which differs from the identity matrix in that one of elementary row operations is performed on the identity matrix. The following matrices are all elementary matrices.

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad 2R1 \text{ on } I_2,$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad R2 \longleftrightarrow R3 \text{ on } I_3,$$

and

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \quad R1 - 3R2 \text{ on } I_2.$$

Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

and let

$$E_1 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, E_3 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{5} \end{pmatrix}, \text{and } E_4 = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix}$$

be elementary matrices. Show that $E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot A = I_2$. If $Au = b$ represents the linear systems of equations in problem 1, then find u in terms of the elementary matrices $E_1, E_2, ..., E_4$.

Solution:

$$E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot A = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{5} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Suppose $Au = b$. Then

$$u = I_2 u = E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot Au = E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot b$$