HOMEWORK SET 3

MAT 217 · FALL 2008

You must show all work to get full credit. You can use a calculator to check your work.

Problem 1. Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 6 & 8 \\ 1 & -3 & -7 \end{pmatrix}$. Find $A^t + B^t, AB^t$, and verify that $(A+B)^t = A^t + B^t$ and $(AB^t)^t = BA^t$.

Solution:

$$A^t + B^t = \begin{pmatrix} 1 & 0 \\ -2 & 4 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 6 & -3 \\ 8 & -7 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 4 & 1 \\ 11 & -2 \end{pmatrix}$$

$$AB^t = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 6 & -3 \\ 8 & -7 \end{pmatrix} = \begin{pmatrix} 16 & -14 \\ 64 & -47 \end{pmatrix}$$

$$(A+B)^t = \left(\begin{pmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 6 & 8 \\ 1 & -3 & -7 \end{pmatrix} \right)^t = \begin{pmatrix} 5 & 4 & 11 \\ 1 & 1 & -2 \end{pmatrix}^t = \begin{pmatrix} 5 & 1 \\ 4 & 1 \\ 11 & -2 \end{pmatrix} = A^t + B^t.$$

$$BA^t = \begin{pmatrix} 4 & 6 & 8 \\ 1 & -3 & -7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 4 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 16 & 64 \\ -14 & -47 \end{pmatrix} = \begin{pmatrix} 16 & -14 \\ 64 & -47 \end{pmatrix}^t = (AB^t)^t$$

Problem 2. Find x, y, z and t if $3 \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix} = 3 \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix} = \begin{pmatrix} x+4 & x+y+6 \\ z+t-1 & 2t+3 \end{pmatrix}$$

It follows that

$$\begin{aligned} 3x &= x+4 \\ 3y &= x+y+6 \\ 3z &= z+t-1 \\ 3t &= 2t+3 \end{aligned}$$

So $2x = -x + 3x = x - x + 4 = 4$. This shows that $x = 2$. Now, $3y = 2 + y + 6$. This

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shows that $3y = y + 8$. And so, $y = 4$. It follows that $3t = 2t + 3$ implies that $t = 3$. Now, $3z = z + 3 - 1$. So $z = 1$.

Problem 3. Find the product of $\begin{pmatrix} x & y & z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\begin{pmatrix} x & y \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

Solution:

$$\begin{pmatrix} x & y & z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x & y & z \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x^2 + y^2 + z^2.$$

$$\begin{pmatrix} x & y \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 3y & 2x + 4y \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = x^2 + 3xy + 2xy + 4y^2 = x^2 + 5xy + 4y^2.$$

Problem 4. Find x, y, z, and t if $\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Find matrix $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$.

Solution:

$$\begin{pmatrix} 3x & 3y \\ x + 2z & y + 2t \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

It follows that

$$\begin{aligned} 3x &= 1 \\ 3y &= 0 \\ x + 2z &= 0 \\ y + 2t &= 1 \end{aligned}$$

It follows that $x = \frac{1}{3}, y = 0, z = -\frac{1}{6}$, and $t = \frac{1}{2}$.