HOMEWORK SET 1

MAT 217 · FALL 2008

You must show all work to get full credit. You can use a calculator to check your work.

Problem 1. Find the solution to the system of equations below using the substitution method

$$\begin{cases} 2x + 3y &= 8 \\ x + 7y &= 15 \end{cases}$$

Solution: Let $x = 15 - 7y$. Then

$$8 = 2x + 3y = 2(15 - 7y) + 3y = 30 - 14y + 3y = 30 - 11y.$$

This shows that $y = 2$. Substituting 2 for y in $x = 15 - 7y$, we have $x = 1$.

Problem 2. Find the solution to the system of equations below by finding the reduced row echelon of the augmented matrix of system of equations.

$$\begin{cases} 4x + 7y &= 56 \\ 2x + 3y &= 30 \end{cases}$$

The augmented matrix of the system of equations is

$$\left(\begin{array}{cc|c} 4 & 7 & 56 \\ 2 & 3 & 30 \end{array}\right).$$

We want to find the reduced row echelon form of this matrix by performing a series of elementary operations. Start by subtracting twice second row from the first; that is, $R1 - 2R2 \to R1$. When we perform this operation, we get

$$\left(\begin{array}{cc|c} 0 & 1 & -4 \\ 2 & 3 & 30 \end{array}\right).$$

We switch the first and second row ($R1 \longleftrightarrow R2$) to get

$$\left(\begin{array}{cc|c} 2 & 3 & 30 \\ 0 & 1 & -4 \end{array}\right).$$

Subtract three times the second row from the first ($R1 - 3R2 \to R1$) to get

$$\left(\begin{array}{cc|c} 2 & 0 & 42 \\ 0 & 1 & -4 \end{array}\right).$$

Divided the first row by 2 ($1/2R1 \to R1$) to get

$$\left(\begin{array}{cc|c} 1 & 0 & 21 \\ 0 & 1 & -4 \end{array}\right).$$

Therefore, $x = 21$ and $y = -4$.

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Problem 3. Let the reduced row echelon form of A be

$$\left(\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 4 \end{array}\right).$$

Determine A if the first and last columns of A are

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 5 \\ 6 \end{pmatrix}.$$

Solution: The matrix A must have the form

$$\left(\begin{array}{cc|c} 1 & b & 5 \\ 2 & d & 6 \end{array}\right).$$

But then the system of linear equations corresponding to A is

$$\begin{cases} x + by &= 5 \\ 2x + dy &= 6 \end{cases}$$

From the reduced row echelon form A, it follows that $x = -3$ and $y = 4$. So

$$\begin{aligned} -3 + 4b &= 5 \\ 2(-3) + 4d &= 6 \end{aligned}$$

This shows that $b = 2$ and $d = 3$. Therefore,

$$A = \left(\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & 3 & 6 \end{array}\right).$$

Problem 4. Let the reduced row echelon form of A be

$$\left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array}\right).$$

Determine A if the first and second columns of A are

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Solution: The matrix A must have the form

$$\left(\begin{array}{cc|c} 1 & -1 & h \\ -2 & 1 & k \end{array}\right).$$

But then the system of linear equations corresponding to A is

$$\begin{cases} x - y &= h \\ -2x + y &= k \end{cases}$$

From the reduced row echelon form A, it follows that $x = 4$ and $y = 3$. So

$$\begin{aligned} 4 - 3 &= h \\ -2(4) + 3 &= k \end{aligned}$$

This shows that $h = 1$ and $k = -5$. Therefore,

$$A = \left(\begin{array}{cc|c} 1 & -1 & 1 \\ -2 & 1 & -5 \end{array}\right).$$