SUPPLEMENTAL PROBLEM

MAT217 · FALL 2008

Cost Minimization. Suppose Green vehicles is under contract to produce and deliver (for a fixed price) 1 million units of hybrid electric cars during the next year. It employs capital (K) and labor (L). If the firm seeks to maximize profits while meeting the terms of the contract, its production decision can be characterized as a constrained cost minimization problem in which the firm chooses the least cost combination of K and L are necessary to produce 1 million units.

Its objective is then to minimize the cost function

$$C(K, L) = K + 3L$$

subject to the output constraint $Q_0 = f(K, L)$, where

$$f(K, L) = 2\sqrt{KL}$$

set $Q_0 = 1$,, where Q_0 equals 1 million units of hybrid electric cars. f is the production function for the firm.

Solution: We begin by deriving the h that is a linear combination of C and f; that is,

$$h(K, L) = C(K, L) - \lambda f(L, K)$$

We now determine the partial derivatives h_K and h_L. And so,

$$\begin{aligned} h_K &= \frac{\partial h}{\partial K} \\ &= 1 - \lambda\sqrt{\frac{L}{K}} \end{aligned}$$

and

$$\begin{aligned} h_L &= \frac{\partial h}{\partial L} \\ &= 3 - \lambda\sqrt{\frac{K}{L}} \end{aligned}$$

We set the partials equal to zero in order to find the critical points. But then,

$$\begin{cases} 1 - \lambda\sqrt{\frac{L}{K}} = 0 \\ 3 - \lambda\sqrt{\frac{K}{L}} = 0 \end{cases}$$

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Using the addition method, we multiply both equations by the factor $\sqrt{KL}$, we find that

$$\begin{cases} \sqrt{KL} - \lambda L = 0 \\ 3\sqrt{KL} - \lambda K = 0 \end{cases}$$

Now multiply equation one by -3 to obtain the following system of equations

$$\begin{cases} -3\sqrt{KL} + 3\lambda L = 0 \\ 3\sqrt{KL} - \lambda K = 0 \end{cases}$$

Now add the two equation to get,

$$\begin{aligned} 3\lambda L - \lambda K &= 0 \\ \lambda(3L - K) &= \end{aligned}$$

And so, $\lambda = 0$ and $K = 3L$ is solution to the system of equations. However, we cannot have the trivial solution $\lambda = 0$. So it must be that $K = 3L$. We substitute $3L$ for L in $f(K, L) = 1$. We obtain

$$\begin{aligned} f(3L, L) &= 1 \\ 2\sqrt{3L \cdot L} &= \\ 2\sqrt{3}L &= \end{aligned}$$

But then, $L = \frac{1}{2\sqrt{3}}$ and $K = \frac{3}{2\sqrt{3}}$. We have a minimum at $(1/2\sqrt{3}, 3/2\sqrt{3})$, and that minimum is $\sqrt{3}$ or \$1.73 million dollars.