

TAKE HOME EXAM I

MAT 217 · FALL 2008

You must show all work to get full credit.

Problem 1. A 5-card poker hand is said to be full house if it consist of 3 card of the same denomination and 2 cards of the same denomination. (That is, a full house is three of a kind plus a pair.) What is probability that one is dealt a full house?

Problem 2. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer and $1 - p$ the probability that the student guess. Assume that a student who guess at the answer will be correct with the probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly? What is probability that a student knew the answer to a question given that he or she correctly answered if $m = 1$ and $p = 0.5$.

Problem 3. The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} 0 & \text{if } x \leq 100 \\ \frac{100}{x^2} & \text{Otherwise} \end{cases}$$

What is the probability that exactly 2 of 5 such tubes in a radio set will have to be the replaced within the first 150 hours of operation? Assume that the events $E_i, i = 1, 2, 3, 4, 5$, that the i th such tube will have to be replaced within this time, are independent.

Problem 4. Find numbers x_1 and x_2 that maximize the sum $x_1 + x_2$ subject to the constraints $x_1 \geq 0, x_2 \geq 0$, and

$$\begin{cases} x_1 + 2x_2 \leq 4 \\ 4x_1 + 2x_2 \leq 12 \\ -x_1 + x_2 \leq 1 \end{cases}$$