

Zeros of a Polynomial

Algebraically solve $f(x) = 0$ for the following equations:

1. $f(x) = x^3 - 7x^2 + 11x$, given that 0 is a zero of $f(x)$.
2. $f(x) = 2x^3 + 3x^2 - 89x + 120$, given that 5 is a zero of $f(x)$.
3. $f(x) = x^3 - 3x^2 - x + 3$, given that 1 is a zero of $f(x)$.

Zeros and Multiplicities

Algebraically find all the zeros and their multiplicities for the following functions:

4. $f(x) = 2x^3 - 3x^2 - 12x + 20$
5. $f(x) = x^3 - 3x + 2$
6. $f(x) = 3x^3 + 22x^2 + 15x - 100$

End Behavior of Polynomials

Use the leading coefficient test to determine the end behavior of the following polynomial functions:

7. $f(x) = -2x^3 + x - 2$
8. $f(x) = -x^{10} - 3x^9 + x^2$
9. $f(x) = 2x^3 + x - 2$

Long or Synthetic Division

Divide the following using long or synthetic division:

10. $\frac{2x^4 - 6x^2 + 1}{x + 1}$
11. $\frac{4x^2 - 8x + 1}{2x - 1}$
12. $\frac{2x^3 - 7x^2 + 2x + 3}{x - 3}$

Vertical Asymptotes

Find the equation of the vertical asymptotes (if any) of the following functions:

13. $f(x) = \frac{3x + 2}{x^2 - 1}$
14. $f(x) = \frac{x + 4}{3x + 1}$
15. $f(x) = \frac{x + 2}{x^2 - 4}$

Applications of Rational Functions

16. The following rational function in hundreds models the population of a certain species of animal, where t is measured in days. What number does the population approach in the long run?

$$p(t) = \frac{10t^3 + 2}{2t^3 + 1}$$

17. The average cost of producing a popular board game is given by the function:

$\bar{C}(x) = \frac{1500 + 15x}{x}$, $x \geq 0$, when x is the number of the board game sold, identify the horizontal asymptote of the function and explain its meaning in this context.

18. The function $N(t) = \frac{0.8t + 100}{5t + 4}$, $t \geq 15$, gives the body concentration $N(t)$, in parts per million of a certain dosage of medication after time t , in hours. Find the horizontal asymptote of the graph and explain the meaning in the context of the problem.

Rewrite in the equivalent logarithmic form

19. $a^{x+1} = 65$

20. $e^{3x} = 5$

Rewrite in the equivalent exponential form

21. $\log_6(4x) = 10$

22. $\ln(B) = A$

Compound interest

23. Find the accumulated value of an investment of \$21,000 at an interest rate of 5.6% for 7 years:

a) compounded monthly

b) compounded continuously

24. a) What initial investment at 3.75% interest compounded continuously for 10 years will accumulate to \$20,000? Round your answer to the nearest cent.

b) What initial investment at 4.25% interest compounded monthly for seven years will accumulate to \$20,000? Round your answer to the nearest cent.

Properties of Logarithms

Use properties of logarithms to write as a sum or difference logarithms with no exponents.

25. $\log\left(\frac{x^5 y^7}{z^3}\right)$

26. $\ln\left((x-1)^{\frac{3}{2}} \sqrt{\frac{(y+3)^4}{z^8}}\right)$

Use properties of logarithms to express the following as a single logarithms

27. a) $2\ln(x) - 5\ln(y) + 9\ln(w)$ b) $\frac{3}{2}\ln(x+3) - \ln(x) - \frac{1}{2}\ln(x+3)$

28. a) $3\log(A) - 4\log(B) + 5\log(C) - 6\log(D)$ b) $\log(8) + \log(x^2 - 1) - \log(x) - \log(x+1)$

Exponential Equations

Solve the following for x

29. a) $2^{2x+17} = 8$ b) $10e^{3x-7} = 5$

30. a) $e^{2x} + 2e^x - 35 = 0$ b) $(7)^{2x} + 2(7)^x - 15 = 0$ c) $2e^{2x} + 3e^x - 20 = 0$

Domain of Logarithms function:

Find the domain of the following function;

31. a) $f(x) = \ln(6 - 2x)$

 b) $f(x) = \log(4x + 16)$

Logarithms Equations

32. Find the x-intercept of the following function:

 a) $f(x) = 4 - 2\log_3(2x - 10)$

 b) $f(x) = \ln(2x + 3)$

33. Solve the following for x :

 a) $\log_6(x) - \log_6(x - 5) = 2$

 b) $\ln(x) + \ln(2x + 1) = 0$

Applications of exponential equations:

34. How long will it take any quantity of iodine 131 to decay to 25% of its initial amount, knowing that it decays according to the function

$A(t) = A_0e^{-0.087t}$, where t is the number of days?

35. The population of Merchantville was 20,000 in 1990 and 25,000 in 1995. If exponential growth is assumed, find a model for the population growth, and then use the model to determine the population in 2008.

36. A sample of 500 grams of radioactive lead 210 decays to polonium 210 according to the function $A(t) = 500e^{-0.032t}$, where t is in years. Find the amount of the sample remaining after

- (i) 4 years, (ii) 8 years. (iii) Find the half-life.

More with polynomials and zeros

37. Identify the zeros and the multiplicities of each zero for $f(x) = -2x^4(x+3)^2(x-7)^8$.
38. Construct a degree 4 polynomial with real coefficients with zeros at $3i$ (multiplicity 1), -4 (multiplicity 2) and with leading coefficient of 1.
39. Construct a degree 3 polynomial with real coefficients, with zeros at $2 + 3i$ (multiplicity 1), 5 (multiplicity 1), and with leading coefficient of 1.

More with rational functions

40. Construct a rational function with the following characteristics:
- x -intercepts at $(2,0)$ and $(7,0)$
 - vertical asymptotes at $x = 4$ and $x = -5$
 - horizontal asymptote at $y = 9$

Answers (some of which have the possibility of not being incorrect)

1. $0, \frac{7}{2} \pm \frac{1}{2}\sqrt{5}$ 2. $5, -8, \frac{3}{2}$ 3. $1, -1, 3$

4. zero at $\frac{-5}{2}$ mult. 1 and zero at 2 mult. 2

5. zero at -2 with mult. 1 and zero at 1 with mult. 2

6. zeros at $-4, \frac{5}{3}, -5$ with mult. of 1 for each.

7. rises left, falls right 8. falls left, falls right 9. falls left, rises right

10. Q: $2x^3 - 2x^2 - 4x + 4$ R: -3 11. Q: $2x - 3$ R: 4

12. Q: $2x^2 - x - 1$ R: 0

13. $x = \pm 1$ 14. $x = -\frac{1}{3}$ 15. $x = 2$

16. 5 hundred

17. 15 is the H.A., the average cost when producing a great number of games is \$15.

18. 0.16 ppm is the concentration after a long time.

19. $\log_a(65) = x + 1$ 20. $\ln(5) = 3x$ 21. $4x = 6^{10}$ 22. $B = e^A$

23. a) \$31,050.37 b) \$31,078.69 24. a) \$13,745.79 b) \$14,861.26

25. $5 \log(x) + 7 \log(y) - 3 \log(z)$

26. $\frac{3}{2} \ln(x-1) + 2 \ln(y+3) - 4 \ln(z)$

27. a) $\ln\left(\frac{x^2 w^9}{y^5}\right)$ b) $\ln\left(\frac{x+3}{x}\right)$

28. a) $\log\left(\frac{A^3 C^5}{B^4 D^6}\right)$ b) $\log\left(\frac{8x-8}{x}\right)$

29. a) -7 b) $\frac{7 - \ln(2)}{3} \approx 2.102$

30. a) $\ln 5 \approx 1.609$ b) $\frac{\ln(3)}{\ln(7)} = \log_7(3) \approx 0.56458$ c) $\ln\left(\frac{5}{2}\right) \approx 0.916$

31. a) $(-\infty, 3)$ b) $(-4, \infty)$ 32. a) $(\frac{19}{2}, 0)$ b) $(-1, 0)$ 33. a) $\frac{36}{7}$ b) $\frac{1}{2}$

34. 15.93 days 35. $P = 20000e^{0.044629t}; 44,658$

36. i) 439.93 gm ii) 387.07 gm iii) $\frac{\ln(2)}{k} = \frac{\ln(2)}{0.032} = 21.66$ years

37. 0, multiplicity 4; -3, multiplicity 2; 7, multiplicity 8

38. $p(x) = (x^2 + 9)(x + 4)^2$

39. $p(x) = (x - 5)(x^2 - 4x + 13)$

40. $r(x) = \frac{9(x-2)(x-7)}{(x-4)(x+5)}$