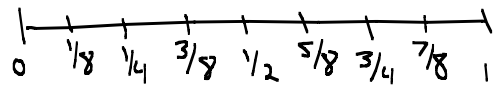
Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate $\int_0^1 \sqrt{1+x^4}\,dx$ with $n=8$.

$$\Delta x = \frac{1-0}{8} = \frac{1}{8}$$



$$T_8 = \frac{\frac{1}{8}}{2}\Big[\sqrt{1+0^4} + 2\sqrt{1+(1/8)^4} + 2\sqrt{1+(1/4)^4} + 2\sqrt{1+(3/8)^4} + 2\sqrt{1+(1/2)^4}$$
$$+ 2\sqrt{1+(5/8)^4} + 2\sqrt{1+(3/4)^4} + 2\sqrt{1+(7/8)^4} + \sqrt{1+1^4}\Big]$$

$$\approx 1.0912708425 7\ldots$$

$$M_8 = \frac{1}{8}\sqrt{1+(1/16)^4} + \frac{1}{8}\sqrt{1+(3/16)^4} + \frac{1}{8}\sqrt{1+(5/16)^4} + \frac{1}{8}\sqrt{1+(7/16)^4}$$
$$+ \frac{1}{8}\sqrt{1+(9/16)^4} + \frac{1}{8}\sqrt{1+(11/16)^4} + \frac{1}{8}\sqrt{1+(13/16)^4} + \frac{1}{8}\sqrt{1+(15/16)^4}$$

$$\approx 1.088508696$$

$$S_8 = \frac{\frac{1}{8}}{3}\Big[\sqrt{1+0^4} + 4\sqrt{1+(1/8)^4} + 2\sqrt{1+(1/4)^4} + 4\sqrt{1+(3/8)^4} + 2\sqrt{1+(1/2)^4}$$
$$+ 4\sqrt{1+(5/8)^4} + 2\sqrt{1+(3/4)^4} + 4\sqrt{1+(7/8)^4} + \sqrt{1+1^4}\Big]$$

$$\approx 1.0894429899\ldots$$

Evaluate $\int_2^{\infty} \frac{dx}{x \ln x}$ or show that it is divergent.

$$\int \frac{dx}{x \ln x} \quad \begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array} \quad = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$$

$$\int_2^{\infty} \frac{dx}{x \ln x} = \lim_{t \to \infty} \int_2^{t} \frac{dx}{x \ln x} = \lim_{t \to \infty} \ln|\ln x| \Big]_2^t$$

$$= \lim_{t \to \infty} \ln|\ln t| - \ln|\ln 2| = \infty$$

The integral is divergent

Evaluate $\int_2^6 \frac{y}{\sqrt{y-2}}\,dy$ or show that it is divergent.

$$\int \frac{y}{\sqrt{y-2}}\,dy \quad \begin{array}{l} u = y-2 \\ du = dy \\ y = u+2 \end{array} \quad = \int \frac{u+2}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}}\,du + \int \frac{2}{\sqrt{u}}\,du$$

$$= \int \sqrt{u}\,du + \int \frac{2}{\sqrt{u}}\,du = \frac{2}{3}u^{3/2} + 4u^{1/2} + C = \frac{2}{3}(y-2)^{3/2} + 4(y-2)^{1/2} + C$$

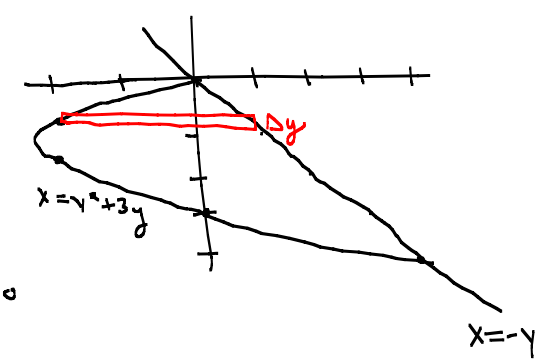
$$\int_2^6 \frac{y}{\sqrt{y-2}}\,dy = \lim_{t \to 2^+} \int_t^6 \frac{y}{y-2}\,dy = \lim_{t \to 2^+} \left[\frac{2}{3}(y-2)^{3/2} + 4(y-2)^{1/2}\right]_t^6$$

$$= \lim_{t \to 2^+} \left[\left(\frac{2}{3}(6-2)^{3/2} + 4(6-2)^{1/2}\right) - \left(\frac{2}{3}(t-2)^{3/2} + 4(t-2)^{1/2}\right)\right] = \frac{40}{3} - 0 = \frac{40}{3}$$

Find the area of the region bounded by $x + y = 0$ and $x = y^2 + 3y$.

$x = -y \qquad -y = y^2 + 3y \Rightarrow 0 = y^2 + 4y \Rightarrow 0 = y(y+4) \Rightarrow y = 0$ or $y = -4$

Intersect at $(0,0)$ $(4,-4)$



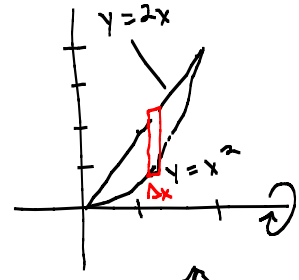
$$A = \int_{-4}^{0} \left((-y) - (y^2 + 3y)\right) dy$$

$$= \int_{-4}^{0} -y - y^2 - 3y\,dy = \int_{-4}^{0} -y^2 - 4y\,dy = -\frac{1}{3}y^3 - 2y^2 \Big]_{-4}^{0}$$

$$= \left(-\frac{1}{3}(-64) - 2(16)\right) - (0) = \frac{32}{3}$$

Find the volume of the solid obtained by rotating the region bounded by $y=2x$ and $y=x^2$ about the x-axis.

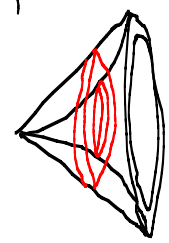
$x^2=2x \Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0 \Rightarrow x=0$ or $x=2$

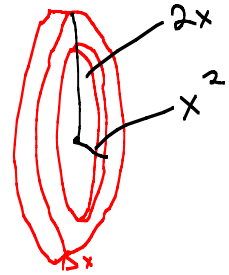


$$V=\int_0^2 \pi\left[(2x)^2-(x^2)^2\right]dx=\pi\int_0^2 4x^2-x^4\,dx$$

$$=\pi\left[\frac{4}{3}x^3-\frac{1}{5}x^5\right]_0^2=\pi\left[\left(\frac{4}{3}(8)-\frac{1}{5}(32)\right)-0\right]$$

$$=\pi\left[\frac{64}{15}\right]=\frac{64\pi}{15}$$



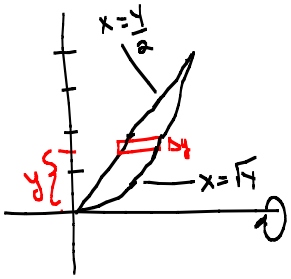


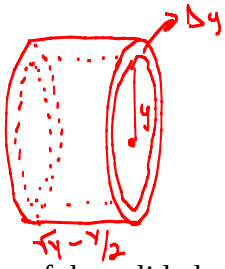
Volume of washer

$\pi\left[(2x)^2-(x^2)^2\right]\Delta x$

with shells: $V=\int_0^4 2\pi(y)\left(\sqrt{y}-\frac{y}{2}\right)dy$

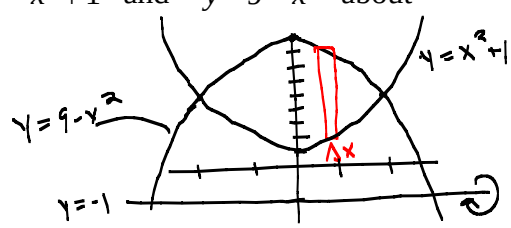
$=\frac{64\pi}{15}$





Find the volume of the solid obtained by rotating the region bounded by $y=x^2+1$ and $y=9-x^2$ about $y=-1$.

$x^2+1=9-x^2 \Rightarrow 2x^2=8 \Rightarrow x^2=4 \Rightarrow x=\pm 2$



$$V=\int_{-2}^{2}\pi\left[\left((9-x^2)-(-1)\right)^2-\left((x^2+1)-(-1)\right)^2\right]dx$$

$$=\pi\int_{-2}^{2}(10-x^2)^2-(x^2+2)^2\,dx$$

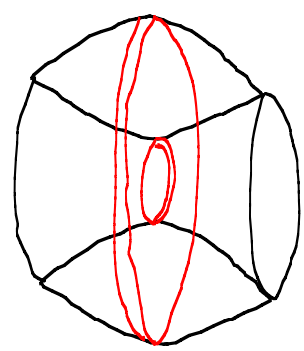
$$=\pi\int_{-2}^{2}100-20x^2+x^4-x^4-4x^2-4\,dx$$

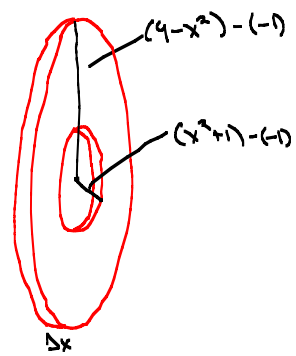
$$=\pi\int_{-2}^{2}96-24x^2\,dx=\pi\left[96x-8x^3\right]_{-2}^{2}$$

$$=\pi\left[\left[96(2)-8(8)\right]-\left[96(-2)-8(-8)\right]\right]$$

$$=\pi\left[(192-64)-(-192+64)\right]$$

$$=256\pi$$





Volume of washer:

$\pi\left[\left((9-x^2)-(-1)\right)^2-\left((x^2+1)-(-1)\right)^2\right]\Delta x$

Set up, but do not evaluate, an integral to find the length of the curve $y=1/6(x^2+4)^{3/2}$ with $0 \le x \le 3$

$$\frac{dy}{dx} = \frac{1}{6} \cdot \frac{3}{2}(x^2+4)^{1/2} \cdot 2x = \frac{x}{2}\sqrt{x^2+4}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4}(x^2+4) = \frac{x^4+4x^2}{4}$$

$$L = \int_0^3 \sqrt{1+\frac{x^4+4x^2}{4}}\, dx$$

Set up, but do not evaluate, an integral to find the length of the curve $y=2(\ln(\sin x/2))$ with $\pi/3 \le x \le \pi$

$$\frac{dy}{dx} = 2\,\frac{1}{\sin(\frac{x}{2})} \cdot \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2} = \frac{\cos(\frac{x}{2})}{\sin(\frac{x}{2})} = \cot\left(\frac{x}{2}\right)$$

$$\left(\frac{dy}{dx}\right)^2 = \cot^2\left(\frac{x}{2}\right)$$

$$L = \int_{\pi/3}^{\pi} \sqrt{1+\cot^2\left(\frac{x}{2}\right)}\, dx = \int_{\pi/3}^{\pi} \csc\left(\frac{x}{2}\right) dx$$

A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?

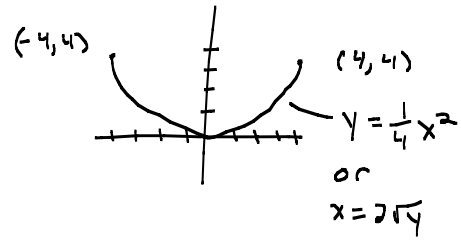
$f(x) = kx$ $\qquad$ $30\,N = k\,(.03\,m)$ $\qquad$ $k = 1000\ N/m$

$20\,cm - 12\,cm = 8\,cm = 0.08\,m$

$$W = \int_0^{0.08} 1000x\, dx = 500x^2\Big]_0^{0.08} = 500\left[(0.08)^2 - 0\right]$$

$$= 3.2\ J$$

A tank full of water has the shape of a paraboloid of revolution (its shape is obtained by rotating a parabola about a vertical axis). If its height is 4 ft and the radius at the top is 4 ft, find the work required to pump the water out of the tank. (Use the fact that water weighs 62.5 lb/ft^3.

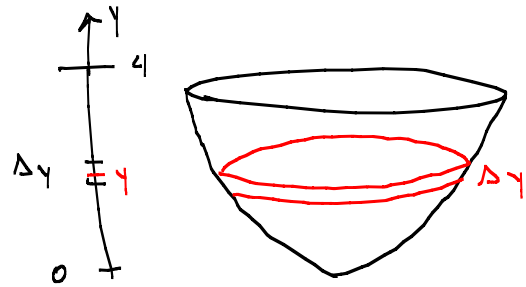


parabola is given by $y=ax^2$

$(4,4)$ is a point on the parabola, so

$4=16a \Rightarrow a=1/4$

$y=\frac{1}{4}x^2 \Rightarrow x^2=4y \Rightarrow x=2\sqrt{y}$



The slice of water is a disk of radius $2\sqrt{y}$ and must be moved $(4-y)$ ft

Volume of the slice:

$\pi(2\sqrt{y})^2\Delta y = 4\pi y\,\Delta y$ ft^3

Weight of the slice

$(4\pi y\,\Delta y)(62.5) = 250\pi y\,\Delta y$ lb

Work needed to move the slice

$(250\pi y\Delta y \text{ lb})(4-y \text{ ft}) = 250\pi(4y-y^2)\Delta y$ ft·lb

So the work needed to pump the water out of the tank is:

$$W=\int_0^4 250\pi(4y-y^2)\,dy = 250\pi\int_0^4 4y-y^2\,dy$$

$$=250\pi\left[2y^2-\frac{1}{3}y^3\right]_0^4 = 250\pi\left[\left(2(16)-\frac{1}{3}(64)\right)-0\right]$$

$$=250\pi\left[32-\frac{64}{3}\right] = 250\pi\left[\frac{32}{3}\right] = \frac{8000\pi}{3} \text{ ft-lb}$$