We studied several combinatorial games this week, all defined in Part I of the game theory text by Thomas S. Ferguson. (See our course web page for a link to the online text.)

Suppose that you are playing the game of 21 chips against a worthy opponent.

1. If it is your turn with 5 chips remaining, what is your move?
2. If it is your turn with 19 chips remaining, what is your move?
3. If it is your turn with 21 chips remaining, what is your move?
4. If it is your turn with 12 chips remaining, why will you lose?

Suppose that you are playing the game of nim against a worthy opponent. Let \((3, 2)\) denote the position with a stack of three chips and a stack of two chips.

5. If it is your turn in position \((2, 1)\), what is your move?
6. If it is your turn in position \((4, 3, 2)\), what is your move?
7. If it is your turn in position \((15, 14, 10, 3, 3, 1)\), what is your move?
8. If it is your turn in position \((10, 7, 7, 5)\), what is your move?
9. If it is your turn in position \((5, 4, 2, 2, 1)\), why will you lose?

Suppose that you are playing the game of chomp against a worthy opponent.

10. If it is your turn in the following position, what is your move?

\[
\begin{array}{cc}
C & C \\
X & C \\
\end{array}
\]
11. If it is your turn in the following position, what is your move?

\[
\begin{array}{ccc}
C & C \\
X & C & C & C
\end{array}
\]

12. If it is your turn in the following position, why will you lose?

\[
\begin{array}{ccc}
C & C & C & C \\
X & C & C & C & C
\end{array}
\]

13. In the game of chomp which has the following starting position, write all legal positions which can occur. (Draw all positions from all games.) Label each position as P-position or N-position.

\[
\begin{array}{cc}
C & C \\
X & C & C
\end{array}
\]