Exercise #1. Determine whether each of these conditional statements is true or false.

(1) If $1 + 1 = 3$, then unicorns exist.
   True. Since statements $1 + 1 = 3$ and unicorns exist are both false, and conditional statement can only be false when the hypothesis condition is true while the conclusion is false, hence this conditional statement is true.

(2) If $1 + 1 = 3$, then dogs can fly.
   True. By the same argument in the first problem.

(3) If $1 + 1 = 2$, then dogs can fly. False. The premise is true, i.e., $1 + 1 = 2$, however, the conclusion that dogs can fly is false, so the conditional statement is false.

(4) If $2 + 2 = 4$, then $1 + 2 = 3$.
   True, since both the premise and conclusion are true.

Exercise #2. Use a truth table to determine if the following formulas are tautologies, contradictions, or neither.

(1) contradiction

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land \neg q$</th>
<th>$\neg p \lor q$</th>
<th>$(p \land \neg q) \land (\neg p \lor q)$</th>
</tr>
</thead>
<tbody>
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Key words and phrases. Conditional, Biconditional, Truth Table, Contrapositive, Converse, Inverse.
Exercise #3. Give the converse, contrapositive, and inverse of the following implications.

(1) If I don’t have homework tonight, then I will go to a movie.
Converse: If I go to a movie tonight, then I won’t have homework.
Inverse: If I have homework tonight, then I won’t go to a movie.
Contrapositive: If I don’t go to a movie, then it means I will have homework tonight.

(2) I go to the beach whenever it is a sunny day.
Converse: Whenever I go to the beach, it is a sunny day.
Inverse: Whenever it isn’t a sunny day, I don’t go to the beach.
Contrapositive: Whenever I don’t go to the beach, it isn’t a sunny day.

(3) A person can be a student at ASU only if they have graduated from high school.
Converse: A person has graduated from high school only if they can be students at ASU.
Inverse: A person can not be a student at ASU only if they have not graduated from high school.
Contrapositive: A person has not graduated from high school only if they can not be students at ASU.
(4) When I stay up late, it is necessary that I sleep until noon.

Converse: When it is necessary that I sleep until noon, then I stay up late.

Inverse: When I don’t stay up late, it is not necessary that I sleep until noon.

Contrapositive: When it is not necessary that I sleep until noon, then I don’t stay up late.

Exercise #4. Let $A, B, C, D$ and $E$ be statements. Suppose the following:

$$B \lor (C \rightarrow E),$$
$$B \rightarrow D,$$
$$\neg D \rightarrow (E \rightarrow A),$$
$$\neg D$$

Prove that $C \rightarrow A$.

Proof:

Suppose all premises are true, by conditional law,

$B \rightarrow D \iff \neg B \lor D$ and

$\neg D \rightarrow (E \rightarrow A) \iff D \lor (E \rightarrow A)$.

By disjunctive syllogism

$\neg D \land (\neg B \lor D) \Rightarrow \neg B$, and

$\neg B \land (B \lor (C \rightarrow E)) \Rightarrow C \rightarrow E$.

By disjunctive syllogism again,

$\neg D \land (D \lor (E \rightarrow A)) \Rightarrow E \rightarrow A$.

Then by transitive law,

$C \rightarrow E \land E \rightarrow A \Rightarrow C \rightarrow A$.

Therefore, $C \rightarrow A$. 