Exercise #1.

Part 1.

1. $r \land \neg p$.
2. $(\neg p \land q) \land r$.
3. $(\neg q \land \neg p) \land r$.

Part 2.

1. It is not the case that both grizzly bears have been seen in the area and hiking is safe on the trail.
2. It is not the case that either grizzly bears have been seen in the area or hiking is not safe on the trail.
3. It is not the case that both grizzly bears have been seen in the area and berries are not ripe along the trail, and hiking is not safe on the trail.

Part 3.

1. Either grizzly bears have not been seen in the area or hiking is not safe on the trail.
2. Grizzly bears have been seen in the area and hiking is not safe on the trail.
3. Either grizzly bears have been seen in the area or berries are not ripe along the trail, and hiking is not safe on the trail.

Exercise #2.

Part 1. Yes. The third and fourth sentences, ‘There may be beasts in both rooms. There may be keys in both rooms,’ repeat information that was already imparted by the first sentence, ‘In each of these rooms there is either a beast or a key to the prison’.

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Key words and phrases. Logical Connectives.
Part 2.

(1) \((K_1 \lor B_1) \land (K_2 \lor B_2)\)
(2) \((K_1 \land B_2) \lor (K_2 \land B_1)\)
(3) \(B_2 \land K_1\)
(4) \(((K_1 \land B_2) \lor (K_2 \land B_1)) \land \neg(B_2 \land K_1)\) \lor \neg((K_1 \land B_2) \lor (K_2 \land B_1)) \land (B_2 \land K_1)\)

Part 3. Yes, the prisoner can be free. We will show this in cases, depending on the truth value of the second statement.

If the statement in front of room 2 is false, then either there is not a beast in room 2 or there is not a key in room 1, that is, \(\neg B_2 \lor \neg K_1\). But then there must be a key in room 2 and a beast in room 1 \((K_2 \land B_1)\) since the statement in front of the first door must be correct. Thus the prisoner is freed if he chooses room 2.

If the statement in front of room 2 is true then there must be a beast in room 2 and a key in room 1, that is, \(K_1 \land B_2\). In addition, the statement in front of room 1 must be false, that is, \(\neg((K_1 \land B_2) \lor (K_2 \land B_1))\). But this means that there is a key in both rooms or a beast in both rooms, which can be written \(\neg(K_1 \land B_2) \land \neg(K_2 \land B_1)\). Specifically, \(\neg(K_1 \land B_2)\), which contradicts the earlier statement of \(K_1 \land B_2\).

Therefore, the statement in front of room 2 must be false, and so the (smart) prisoner chooses room 2.