A Mathematical Comparison of Prevention Strategies for Addicted Women

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Abstract

Crack-cocaine use among pregnant women is major public health concern leading to children born addicted to cocaine, an increased rate of HIV infection and many other health and social problems. Many programs exist that focus on the rehabilitation of women and mothers who use cocaine. We use deterministic approach to model the effectiveness of these programs. The focus will be on populations of women (often commercial sex workers) who are encouraged or forced to use drugs by drug dealers, pimps or both. The impact of drug rehabilitation and other treatment programs among particular groups is explored as well as the role of drug enforcement on the dynamics of this system. In particular, the roles of pimp’s pressure on women to use drugs, the inability of drug users to quit due to addiction, and the relapses among those in rehabilitation programs are explored. The effect of longer jail terms for drug dealers and pimps is discussed in the context of the model and data available.
1 Introduction

Substance abuse, in general, during pregnancy is a serious public-health issue, consuming valuable health-care resources and contributes to high infant mortality and morbidity. Data indicates an increasing trend in substance abuse among women during pregnancy. The consequences of drug use among pregnant women and their children are multiple. Mothers who smoke during pregnancy may increase the risk that their child will have autism [2]. Marijuana users give birth to babies who are three ounces lighter and $\frac{1}{3}$ inch shorter than babies born to women who do not use marijuana [1].

However, cocaine abuse is far worse. It can increase the risk of hemorrhaging and premature delivery, threatening the lives of both the mother and child. Cocaine users give birth to babies with low average birth weight, more than 5.5 pounds less than babies of women who do not use cocaine [6]. It is estimated that the national cost to care for a “crack baby” is about 3 billion dollars [5]. Head size is often smaller in infants exposed to narcotics. While growth erases some of the physical differences, there may be subtle, long-term deficits in mental or neurological functioning in infants exposed to drugs in the womb. Women who use intravenous drugs or share drugs are at a higher risk of getting deadly diseases like AIDS. Scientists are just beginning to explore how various drugs may effect the development of physical coordination, language and emotional interactions.

A survey found that an estimated 113,000 white women, 75,000 African-American women, and 28,000 Hispanic women use illicit drugs during pregnancy [3]. At some point during their pregnancy, 20.4%, or 820,000, pregnant women smoked cigarettes and 18.8%, or 757,000, drank alcohol [3]. Among those women who used both cigarettes and alcohol, 20.4% also used marijuana and 9.5% took cocaine [3]. African-American women had the highest rates of cocaine use, mainly “crack”, during pregnancy [3]. About 4.5% of African American, 0.4% of white women and 0.7% of Hispanic women use crack-cocaine [3]. The researchers estimate that each year, as many as 375,000 infants may be affected by their mothers’ drug use [1].

Research indicates that women can become addicted quickly to certain drugs, even after casual or experimental use and more than 4 million need treatment [4]. Many of these women do not seek treatment because they are afraid they will not be able to keep or care for their children. Many fear reprisal from their partners or punishment from authorities in the community. Also, many programs have refused to accept pregnant women or have been unable to provide them services they need including prenatal care, parenting skills instruction, childcare and transportation. Many women report
that their drug-using male sex partners initiate them into drug abuse and then sabotaged their efforts to quit [4].

In this project we create a model that examines the influence of a program that effectively educates pregnant and non-pregnant women about the dangers and consequences of drug abuse. We do not prescribe how to create such a program, but rather explore the possibilities should such a program exist. We focus our research on crack-cocaine abuse among women; how we can reduce their rate of drug use and the impact of encouraging pregnant and non-pregnant women to go through rehabilitation. This project is in the context of a system driven by a population of males who use the power and influence of drug addiction to manipulate women. We show that we cannot eliminate drug use among women unless we include both incarceration for drug dealing men and rehabilitation for men and women.

Our paper is organized as follows: section 2 introduces a basic deterministic model where we consider only rehabilitation for men and women; section 3 compares a stochastic version of our model with the deterministic model from section 2; section 4 introduces drug induced mortality to our basic model; section 5 replaces the rehabilitation class from section 4 with a jail class for the males to explore the effects of incarceration; section 6 combines a jail class with a rehabilitation class for men to see how these two factors can reduce the population of drug using women; in section 7 we discuss the results of our models; our conclusions are drawn in section 8; finally section 9 states what we have left for future work.

2 The basic Model

A basic model is introduced to study the impact of education on drug abuse in pregnant and non-pregnant women. The two-sex system consists of nine nonlinear differential equations and is driven by a three dimensional subsystem (the males). The first model (see Figure 1) does not incorporate drug-induced mortality explicitly as it assumes that the average residence time in the system for males is $\frac{1}{\mu_m}$ and for females is $\frac{1}{\mu_f}$. Furthermore, we are assuming that $\mu_m = \mu_f = \mu$. This is of course not true but the conditions will be relaxed later on.

The assumption of equal exit rates let us normalized the system, letting $X = \frac{S_m}{N_m}, Y = \frac{D_m}{N_m}, Z = \frac{R_m}{N_m}, P = \frac{S_f}{N_f}, Q = \frac{D_f}{N_f}, R = \frac{R_f}{N_f}, S = \frac{P_s}{N_f}, T = \frac{P_d}{N_f},$ and $U = \frac{P_r}{N_f}$. We arrive at the following system of equations:
Figure 1: The Deterministic Model
\[
\begin{align*}
\frac{dX}{dt} &= \mu - \beta_m XY - \mu X, \\
\frac{dY}{dt} &= \beta_m XY + \rho_m Z - (\gamma_m + \mu) Y, \\
\frac{dZ}{dt} &= \gamma_m Y - (\rho_m + \mu) Z, \\
\frac{dP}{dt} &= \mu + \lambda_1 S - \beta_f PY - (\mu + \phi_1) P, \\
\frac{dQ}{dt} &= \beta_f PY + \rho_1 R + \lambda_2 T - (\mu + \phi_2 + \gamma_1) Q, \\
\frac{dR}{dt} &= \gamma_1 Q + \lambda_3 U - (\rho_1 + \phi_3 + \mu) R, \\
\frac{dS}{dt} &= \phi_1 P - \beta_f SY - (\lambda_1 + \mu) S, \\
\frac{dT}{dt} &= \beta_f SY + \phi_2 Q + \rho_2 U - (\lambda_2 + \gamma_2 + \mu) T, \\
\frac{dU}{dt} &= \gamma_2 T + \phi_3 R - (\rho_2 + \lambda_3 + \mu) U,
\end{align*}
\]

where \( X + Y + Z = 1 \) and \( P + Q + R + S + T + U = 1 \).

Here, \( P \) represents the proportion of susceptible women, \( Q \) the proportion of drug using women, \( R \) the proportion of women in rehabilitation, \( S \) the proportion of susceptible pregnant women, \( T \) the proportion of drug using pregnant women, \( U \) the proportion of pregnant women in rehabilitation, \( X \) the proportion of non-drug using men, \( Y \) the proportion of drug using men, and \( Z \) the proportion of men in rehabilitation.

For the male population, \( \mu_m \) denotes the standard mortality or exit rate for all classes. The rate that men enter the susceptible class is equal to the number of men that have died from all classes, thus making the population constant. Once in the susceptible class, men are influenced by other drug-using men to use crack-cocaine via a mass action rate. That is to say, the rate that men cause other men to use drugs is proportional to the density of drug using men in the population. Then, drug using men can go into rehabilitation by a rate \( \gamma_m \). Once in rehabilitation, the men can either stay in rehabilitation or relapse at a rate \( \rho_m \). Note that men or women cannot go from the rehabilitation class back into the susceptible class. This is because we are assuming that addiction to crack-cocaine is very strong and once someone has become addicted, they always have a chance of falling back into addiction. There is no “recovery” from crack-cocaine addiction, similar to what happens with alcoholism.
For the females system, there is also a standard death rate \( \mu_f \) and the number of women that enter the susceptible class is equal to the number of women that die, thus keeping the population size constant. We see that pregnant and non-pregnant susceptible women are influenced by men to use crack-cocaine by a mass incidence rate, however we assume women do not cause other women or men to use crack-cocaine. Often drug using men convince women to use drugs so that the men have better control over the women. This is especially true when the woman is closely associated (married to or living with) a drug using male [4] like the case of pimps or drug dealers and commercial sex workers. In all classes of non-pregnant women \((P, Q, R)\), we see that they can get pregnant at some rate \( \phi_i \). These parameters were approximated by taking the proportion of women who are pregnant in each class relative to the total number of women who are either susceptible, drug using, or in rehabilitation [3, 17, 16, 13, 9]. In all classes of pregnant women \((S, T, U)\), they can have miscarriages or give birth earlier and thus return to a state of non-pregnancy at rate \( \lambda_i \). For the purposes of our models, we only consider a “miscarriage” to occur when the women loses her child after she realizes she is pregnant. Furthermore, we assumed that women cannot get pregnant again until a month after they give birth, have an abortion or miscarriage and that the rate of miscarriage for women who have never used drugs is negligible compared to that of drug using women [6, 8]. When both pregnant and non-pregnant women are in the drug using class, they can go to rehabilitation at a rate \( \gamma_i \). Once in rehabilitation, they can relapse back into the drug using class at a rate \( \rho_i \). Note that we did not include an education program for men because we are assuming that women receive more social support than men from a variety of sources such as families, friends and coworkers and that women are more likely to maintain a social network and engage in treatment [19].

This system can be directly applied to the population of drug dealers, pimps, and commercial sex workers. Male drug-dealers and pimps can cause other males and females to become addicted to crack-cocaine, but female sex workers generally do not cause other women or men to abuse crack-cocaine. Pimps and drug dealers will often sabotage a female sex worker’s attempt to get out of the drug culture so that he can remain in control, and thus the male population is the driving force on this system of crack-cocaine abuse.

2.1 Definition of the basic reproductive number \( R_0 \)

To evaluate our system of equations we analyze the basic reproductive number, \( R_0 \) of drug-abuse, interpreted in epidemiological models as the average number of secondary cases caused by a drug using male. In our system, \( R_0 \) represents the average number of man and women, pregnant or non-pregnant,
Table 1: Parameter List

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>natural mortality rate</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>rate at which women that are not using drugs become pregnant</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>rate at which women that are using drugs become pregnant</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>rate at which women that are in education (rehabilitation) become pregnant</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>rate at which women are susceptible to become pregnant again</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>rate of which women drug users are susceptible to become pregnant again</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>rate of which women in education (rehabilitation) are susceptible to become pregnant again</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>contact rate of women with drug using men</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>rehabilitation rate of non-pregnant women</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>rehabilitation rate of pregnant women</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>relapse rate of non-pregnant women</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>relapse rate of pregnant women</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>relapse rate of men</td>
</tr>
<tr>
<td>$d$</td>
<td>drug induced mortality for men</td>
</tr>
<tr>
<td>$w$</td>
<td>drug induced mortality for women</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>contact rate of men with drug using men</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>rehabilitation rate of men</td>
</tr>
</tbody>
</table>

coerced to use drugs by men at the beginning of the drug epidemic.

To calculate $R_0$, we consider the drug-abuse free equilibrium. The $R_0$ of our system is derived from the male equations, because they are the only group generating secondary cases of drug addiction. In section 2.2, we show that $R_0$ is given by:

$$R_0 = \frac{\beta_m (\rho_m + \mu)}{\mu (\gamma_m + \mu + \rho_m)}$$

and illustrate its role in the stability of the drug free and endemic drug abuse equilibria.

2.2 Calculation of drug-abuse free equilibrium

One possible end state for this model is the drug-abuse free equilibrium. The drug-abuse free equilibrium of the male system is given by $(X^*, Y^*, Z^*) = (1, 0, 0)$, in which the entire population is susceptible but there is no drug
abuse and no one in the rehabilitation class.

We have the following theorem:

**Theorem 2.1.** Let \( \vec{x}_\infty(DF) = (1, 0, 0) \) be the disease free equilibrium of (1) - (3). The system is locally asymptotically stable if and only if \( R_0 < 1 \).

**Proof.** The Jacobian given from the linearization at this equilibrium is:

\[
J(1, 0, 0) = \begin{bmatrix}
-\mu & -\beta & 0 \\
0 & \beta_m - \mu - \gamma_m & \rho_m \\
0 & \gamma_m & -(\rho_m + \mu)
\end{bmatrix}.
\]

Since \(-\mu\) is an eigenvalue, we only need to consider the trace and the determinant of

\[
A = \begin{bmatrix}
\beta_m - \mu - \gamma_m & \rho_m \\
\gamma_m & -(\rho_m + \mu)
\end{bmatrix}.
\]

The \( \det(A) > 0 \iff R_0 < 1 \) and this implies that the \( \text{trace}(A) < 0 \)

The drug abuse free equilibrium is stable for the males. Then in the limit that \( Y \) approaches it’s equilibrium value, we can treat it as a constant and thus linearize the female equations. Doing so, we can calculate the drug abuse free equilibrium as:

\[
P^* = \frac{\lambda_1 + \mu}{\lambda_1 + \mu + \phi_1}, \quad S^* = \frac{\phi_1}{\lambda_1 + \mu + \phi_1}.
\]

### 2.3 Endemic Equilibrium and Stability Analysis

The nonzero solutions of the normalized system are: \( X^* = \frac{1}{R_0}, Y^* = \frac{\mu}{\beta_m}(R_0 - 1), \) and \( Z^* = \frac{\gamma_m \mu}{\beta_m(\rho_m + \mu)}(R_0 - 1) \).

From the Jacobian matrix at \( \vec{x}_\infty(EE) \) we obtained:

\[
J(\vec{x}_\infty) = \begin{bmatrix}
-\mu(R_0 - 1) - \mu & -\frac{\beta_m}{R_0} & 0 \\
\mu(R_0 - 1) & \frac{\beta_m}{R_0} - \mu - \gamma_m & \rho_m \\
0 & \gamma_m & -(\rho_m + \mu)
\end{bmatrix}.
\]

Solving for the eigenvalues of \( J \):
\[
(J - \lambda I) = \begin{bmatrix}
-\mu(R_0 - 1) - \mu - \lambda & -\frac{\beta_m}{R_0} & 0 \\
\mu(R_0 - 1) & \frac{\beta_m}{R_0} - \mu - \gamma_m - \lambda & \rho_m \\
0 & \gamma_m & -(\rho_m + \mu) - \lambda
\end{bmatrix}
\]
\[
= -(\mu + \lambda) \begin{bmatrix}
1 & 1 & 1 \\
0 & \beta_m - \mu - \gamma_m - \lambda + \mu(R_0 - 1) & \rho_m + \mu(R_0 - 1) \\
0 & \gamma_m & -(\rho_m + \mu + \lambda)
\end{bmatrix}
\]
Since \(-\mu\) is an eigenvalue, we only need to consider the trace and the determinant of:
\[
A_E = \begin{bmatrix}
\beta_m - \mu - \gamma_m - \lambda + \mu(R_0 - 1) & \rho_m + \mu(R_0 - 1) \\
\gamma_m & -(\rho_m + \mu + \lambda)
\end{bmatrix}
\]
The \(\text{det}(A_E) > 0\) and \(\text{trace}(A_E) < 0\). This implies that the endemic solution is locally asymptotically stable.

2.4 Numerical Simulations

In the numerical simulations we vary parameters such as the contact rate of men and women and the rehabilitation rate of pregnant and non-pregnant women to see what effects these parameters would have on the proportion of pregnant women that used drugs. We do this to determine how effective our education program would have to be in order to attain a certain amount of success defined by a level of decrease in the number of pregnant women that abused crack-cocaine. In this way we model an education program by altering the parameters that cause pregnant women to abuse crack-cocaine.

We analyze the effects of the interaction of women with drug-using men, see appendix Figure (6). In Figure (b) the value of \(\beta_m\) is decreased from 0.0714 to 0.0414 and in (a) the value of \(\beta_m\) is increased to 0.0914. When comparing the (a) and (b), as we increase the value of \(\beta_m\) we get a correspondingly larger number of drug using women. This shows a direct correlation between the number of drug using women and the rate that women interact with drug using men.

We also analyze the situation of drug using women going into rehabilitation, see appendix Figure (7). In (b) the value of \(\gamma_1\) and \(\gamma_2\) is decreased to 0.001108 and 0.004967. In (a) the value of \(\gamma_1\) and \(\gamma_2\) is increased from 0.007108, 0.014967 to 0.009108 and 0.054967 respectively. We see that as the
value of $\gamma_i$ is increased, there will be more people in rehabilitation. When the value of $\gamma_i$ increases, the effectiveness of the education program for women increases and vice versa.

3 Model with Drug Induced Mortality

In this section we want to analyze the original model after we include drug induced mortality.

\[\text{Male System} \]

\[\begin{array}{c}
D_m \\
\downarrow (\mu+\delta)U_m
\end{array}\]

\[\text{Female System} \]

\[\begin{array}{c}
D_f \\
\downarrow (\mu+\delta)U_f \\
\uparrow (\mu+\delta)F_s
\end{array}\]

Figure 2: The Drug Induced Mortality Model

We have the following theorem for this model:

**Theorem 3.1.** Let \( \bar{x}_\infty(DF) = (1, 0, 0) \) be a disease free equilibrium of (2). The system is locally asymptotically stable if and only if \( R_0 < 1 \) where

\[ R_0 = \frac{\beta_m (\rho_m + \mu)}{\mu (\gamma_m + \rho_m + \mu) + d (\rho_m + \mu)} \]

we compare the new system to the one without drug induced mortality by numerical solutions with the same initial conditions and parameters. We analyze the situation of drug-using women going to rehabilitation, see appendix Figure (7). In (d) the value of $\gamma_1$ and $\gamma_2$ is decreased to 0.001108 and 0.004967 respectively and in (c) the value of $\beta_m$ is increased to 0.1014. If we compare it with (a) and (b), we also see the same behavior at a different scale.
4 Model with Jail Term, No Rehabilitation for Men, and Drug Induced Mortality

We now add a different stage in the male population and include drug induced mortality for men and women where both are different due to the fact that we are assuming that men have a higher risk of getting killed or dying through drug use. For the women we assume that this rate is equal for both pregnant and non-pregnant women. This new approach to the model tells us approximately how long an individual (male) who uses drugs has to remain in jail in order to prevent women from using drugs. Next is a diagram of our model.

Figure 3: Deterministic model with jail term and no rehabilitation in men population

Again we focus on the male equations which are the driving force of the system. The disease free equilibrium is \((X^*, Y^*, Z^*) = (1, 0, 0)\), and to find its stability we looked at the Jacobian matrix. In this case our population is not constant, hence we cannot reduce it to a two dimensional system. Nevertheless we looked at the \(3 \times 3\) Jacobian matrix where \(-\mu\) is an eigenvalue where \(\mu > 0\), and we have the following theorem:

**Theorem 4.1.** Let \(x_\infty(DF) = (1, 0, 0)\) be a disease free equilibrium of (13) - (15) then it is locally asymptotically stable if and only if \(R_0^J < 1\) where

\[
R_0^J = \frac{\beta_m (\rho_m + \mu + d)}{\mu (\gamma_m + \rho_m + \mu + d)}. \tag{16}
\]

In the deterministic simulations we fixed some parameters; \(\beta_f = 0.0714\), \(\lambda_1 = 0.8\), \(\lambda_2 = 0.7\), \(\lambda_3 = 0.75\), \(\gamma_1 = 0.007108\), \(\gamma_2 = 0.014967\), \(\phi_1 = 0.028489\),
\[ \phi_2 = 0.023313, \ phi_3 = 0.049089, \rho_1 = 0.22, \rho_2 = 0.01, \mu = 0.00004, \ d = 0.01, \]
and \[ w = 0.05. \] We will vary \( \beta_m, \gamma_m, \) and \( \rho_m. \) Our initial conditions are:
\[ D_f(0) = 977, \ P_d(0) = 20, \) and \[ D_m(0) = 1840. \] Our starting populations are \[ N_m = 10000, \) and \[ N_f = 10000. \]

In appendix Figure (8,9) we see as the number of men in jail increases the number of men using drugs also increases, which tells us that even if we send men to jail for a significant amount of time they will be replaced by other drug users. We also notice the number of women using drugs increasing but as soon as there are not enough men using drugs the number of women in rehabilitation (education) increases significantly. What is somehow surprising is that the number of pregnant women using drugs does not increase. Based on our model we can say that men do not have as large an impact on pregnant women as they do on non-pregnant women.

In appendix Figure (10,11) when we decrease \( \beta_m \) and we see that there is only a slight outbreak of women using drugs, pregnant or non-pregnant, and there is a noticeable increase in women in rehabilitation. In this case \( R_0^I < 1 \) so eventually there will be no one using crack cocaine which is not realistic. However we can still gain some insight since if the goal is to get rid off crack cocaine abuse, we must take dramatic measures which we will discuss further in our conclusions.

In appendix Figure (12,13) we doubled the sentence for individuals sent to jail for crack cocaine possession which is 10 years [18]. By varying \( \rho_m \) from 10 to 20 years we see a decrease in the number of men using drugs but increases slowly with time. In contrast, the number of men sent to jail increases exponentially. In the case of women there is only a slight increase and decreases slowly which indicates that even when \( R_0^I > 1 \) it is still possible to reduce the number of drug using women by having prosecuted men stay in jail longer.

5 Jail Term and Rehabilitation in Men

As seen in previous sections, based on our model and reality it is practically impossible to make drug abuse disappear. Our model that included jail term in men and no rehabilitation told us that jail is not enough to lower the endemicity of drug use. Hence, if we want to lower the number of men using crack cocaine there are a number of factors that have to be taken into consideration. By previous analysis, keeping men “crack-free” is one way to prevent drug abuse in pregnant and non-pregnant women. In reality, it is difficult to keep men or in fact anyone off crack cocaine. We show in this
Figure 4: Deterministic model including jail term and rehabilitation in the male population

section that by keeping a small proportion of the male population off of drug use, we can lower the number of women who abuse crack-cocaine. The model is normalized and the new variable is \( W = \frac{R_m}{N_m} \).

For the stability analysis we looked at the Jacobian matrix at the disease free equilibrium and we have the following theorem:

**Theorem 5.1.** Let \( \bar{x}_\infty(DF) = (1, 0, 0) \) be a disease free equilibrium of (22) - (24) then is locally asymptotically stable if and only if \( R_0^{jr} < 1 \) where

\[
R_0^{jr} = \frac{\beta_m (\rho_m + \mu + g)}{\mu (\rho_m + \mu + g) + d (\mu + \rho_m) + g (\gamma_m + d)}
\]

Looking at the appendix Figures (14,15) we see that there is a peak in the number of women and men who abuse crack-cocaine. This peak occurs after the first 40 years and slowly drops off. Looking at the male population, we notice that the number of men in rehabilitation will eventually overtake the number of men in jail, but only after 100 years, and overtake the number of drug using men after 150 years.

If we were to double the rate that men go into rehabilitation as in appendix Figures (17,18), then we notice that there is negligible change to the female system. In the male system, there are no qualitative changes, but the number of drug using men and the number of men in jail do decrease. The number of men in rehabilitation also increases substantially. The number of years it takes for the men in rehabilitation to overtake the number of men in jail decreases to approximately 60 years, and 100 to overtake the number of men abusing crack-cocaine.
Looking at appendix Figures (15,18), we notice a curious phenomenon. Although $R_0$ is greater than one, the number of drug using men seems to drop off to very low levels. Out of purely mathematical curiosity, if we increase the time scale of our analysis, then we notice very interesting behavior in Figure (16). There is periodic damped oscillations of the number of recovered men, and periodic spikes in the number of drug using men and the number of men in jail that decreases in amplitude as a function of time. The number of recovered men decreases slowly with time, then increases sharply for a short time, then repeats. Then number of men in jail or using drugs exists at low levels, then experiences sharp peaks right after the number of recovered men reach a relative minimum and jump up to a new relative maximum. The dynamics of this motion need further investigation and is left for future work.

6 Discussion

There will always be a certain level of drug use in the population according to our model and parameters. Having a successful education program would mean altering many of the parameters that cause women to use drugs. Looking at our deterministic model, when we vary $\beta_f$ or $\beta_m$, the number of women who use drugs in the short run changes significantly, but reaches a steady state in the long run. However, our model is not very accurate in the long run because we assume a constant population. If we increase $\beta_f$ by a factor of 10, then we assume that our educational program is failing and that men are getting better at causing women to use drugs. With this assumption, in ten years, the number of women who use drugs increases by 280%, and in 20 years the number of women who use drugs increases by 400%, see appendix Figure (19). If we decrease $\beta_f$ by a factor of 10, then our educational program is increasing the awareness of the detrimental effects of crack-cocaine to women in general. Under these conditions, the number of women who use drugs decreases by 20% in the first ten years, and 44% after 20 years.

It is interesting to note that if we increase $\beta_f$ by a factor of 100, then the number of drug users increases by nearly 1000% in 20 years, but if we decrease $\beta_f$ by a factor of 100, in 20 years the number of drug users decreases by 40%. Clearly, we see that while it is worthwhile to educate women in general and try to decrease the rate that men cause women to use drugs, it is not effective to spend a lot of resources trying to educate the general public about crack-cocaine. Although this does not mean that we should not make an effort to educate the public. Our data clearly shows that if we allow crack-cocaine to be spread more easily, we will have an explosion of drug abuse.
Considering just the population of pregnant women, if we increase $\beta_f$ by a factor of 10 then, in the first ten years the number of susceptible drug using women will decrease by 15%, the number of crack-cocaine using women increased by 757%, and the number of women in rehabilitation decreased by 25%, see appendix Figure (20). In twenty years, the number of susceptible drug using women will decrease by 40%, the number of crack-cocaine using women increased by 550%, and the number of women in rehabilitation does not change. Similarly, if we decrease $\beta_f$ by a factor of 10, then in the first ten years the number of susceptible drug using women will increase by 1.5%, the number of crack-cocaine using women decreased by 43%, and the number of women in rehabilitation decreased by 25%. In twenty years, the number of susceptible drug using women will increase by 4.4%, the number of crack-cocaine using women decreased by 75%, and the number of women in rehabilitation decreases by 20%. However, changing $\beta_f$ by two orders of magnitude makes little difference in the short run, with less than a percent difference from those values obtained by changing $\beta_f$ by only one order of magnitude.

If we look at the men, changing $\beta_m$ produces the same qualitative behaviors observed in the case with the female population, see appendix Figure (21).

In the case with pregnant women, if we decrease $\gamma_i$ by a factor of 10 then there are virtually no pregnant women in rehabilitation during our time scale. If we increase $\gamma_i$ by a factor of 10, then there is a 600% change in the number of pregnant women in rehabilitation, see appendix Figure (22). Thus the population of pregnant women is very sensitive to the $\gamma_i$.

However, the population of pregnant women is small relative to the total population of women. Therefore our deterministic model is relatively insensitive to changes in $\gamma_i$. If we decrease $\gamma_i$ by two orders of magnitude, there is less than a 1% change for the first 60 years, see appendix Figure (23). If we increase $\gamma_i$ by a factor of 10, then it takes over 30 years before we get a 10% difference in the number of women who will abuse crack-cocaine.

Another important parameter to any rehabilitation program is the rate of relapse. Ideally, one would want women to never use drugs again after finishing a rehabilitation program. In the case of pregnant women, we found that if we increased $\rho_1$ and $\rho_2$ by a factor of 100, the number of women in rehabilitation decreases by 20% in ten years, see appendix Figure (24). This is due to the direct effect of women leaving rehabilitation programs. However the total number of crack-cocaine using women changes by only 1%, see
Figure 5: Percent change due to variation of parameters: $\beta_f$ decreased by a factor of 10, $\gamma_i$ increased by a factor of 10, $\rho_i$ decreased by a factor of 10

<table>
<thead>
<tr>
<th>Population</th>
<th>$\beta_f$</th>
<th>$\gamma_i$</th>
<th>$\rho_i$</th>
<th>$\beta_f$ &amp; $\gamma_i$</th>
<th>$\beta_f$ &amp; $\rho_i$</th>
<th>$\gamma_i$ &amp; $\rho_i$</th>
<th>$\beta_f$ &amp; $\gamma_i$ &amp; $\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_s$</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
<td>0.17</td>
<td>0.019</td>
<td>0.15</td>
</tr>
<tr>
<td>$P_d$</td>
<td>-0.47</td>
<td>-0.27</td>
<td>0</td>
<td>-0.67</td>
<td>-0.53</td>
<td>-0.4</td>
<td>-0.73</td>
</tr>
<tr>
<td>$P_r$</td>
<td>-0.6</td>
<td>6</td>
<td>0.8</td>
<td>3</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>$D_f$</td>
<td>-0.44</td>
<td>-0.2</td>
<td>0.04</td>
<td>-0.52</td>
<td>-0.44</td>
<td>-0.28</td>
<td>-0.6</td>
</tr>
<tr>
<td>$S_f + R_f$</td>
<td>0.18</td>
<td>0.05</td>
<td>0.03</td>
<td>0.20</td>
<td>0.18</td>
<td>0.11</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Appendix Figure (25). This is because the number of drug using women is large relative to the number of women in rehabilitation. If we decrease $\rho_1$ and $\rho_2$ by a factor of 100, then the number of pregnant women in rehabilitation programs increases by 40% in ten years. Likewise, the number of drug using women changes by only 2%. In 20 years, if you decrease $\rho_1$ and $\rho_2$ by a factor of 100, then the number of women in rehabilitation increases by 94%, and the number of drug using women decreases by 3%. If you increase $\rho_1$ and $\rho_2$ by a factor of 100, then the number of women in rehabilitation increases by 17% and the number of drug using women increases by 3%.

Although there are other parameters that may have an impact on the dynamics of our model, namely $\lambda_i$ and $\phi_i$, they will be considered as constants in our system. The mechanics behind altering $\lambda_i$ involves changing the gestation period of women or the rate that they have miscarriages and is outside of the scope of our project. Adjusting $\phi_i$, the rate that women get pregnant is another consideration that is outside the scope of our project.

The three most important parameters are $\beta_f$, $\gamma_i$ and $\rho_i$. We have already seen the changes that result from altering just one parameter at a time. The next step is to see whether or not altering two or all three parameters can cause significant changes:

From Figure (5) we see that changing $\beta_f$ is the most effective way to reduce the amount of drug abuse in our model with $\gamma_i$ as the next most effective parameter and $\rho_i$ as the least effective. Combining parameters is always beneficial, but sometimes the amount of change achieved is not much compared to changing just one variable. For example, changing $\beta_i$ causes an 18% reduction in the number of drug free women while changing $\beta_i$ and $\gamma_i$ only creates a 19% reduction and changing $\beta_i$ and $\rho_i$ does not cause significant change. While it is clear that $\beta_i$ is the most important parameter, changing all three parameters brings about the most change.

Changing $\beta_m$, $\gamma_m$ and $\rho_m$ brings about a similar quantitative percentage
change for the males, but our discussion is limited to the female case. We assume that women are much more susceptible to efforts to keep them off crack-cocaine and can benefit the most from such efforts. In addition, changing these parameters will only have marginal effects on women because they will be the object of secondary aid due to the interaction with a reduced $D_m$. If the parameters $\beta_i$, $\gamma_i$ and $\rho_i$ are changed, then that makes women the primary target of aid and they receive the most benefit.

Comparing our models with and without drug-induced death produced very different results. In the case of without drug-induced death, we never reached a steady state in the short run, the population of drug users is constantly increasing. With drug-induced death, the number of drug users would often peak within the first 20 years, and quickly reduce to a steady level within the next hundred years. Although our model is not very accurate in the long term (population growth and immigration would need to be considered), it is interesting to analyze the deterministic model in this time scale and see its behavior. It is clear from these simulations that the endemic solution exists at a much lower level than in the case without drug-induced death sometimes half of what we would get if we did not consider drug induced death after the first ten years, see appendix Figures (7a,c). This qualitative phenomena is paralleled in the case with women in rehabilitation and pregnant women in both classes. Death through cocaine abuse is a very serious problem, with nearly 20,000 people a year dying through drug related causes [20]. Our data indicates that including drug-induced death is significant in that it changes the dynamics of our system and makes it more accurate to real life.

Looking at the case where men go to jail instead of rehabilitation, we see that the dynamics are similar to the case with rehabilitation, but the system is slower. That is, it takes a longer amount of time for the number of drug users to peak and fall off to a steady state by a factor of 2. Also, because it takes longer for the number of drug users to peak, there are a larger number of drug users at the peak.

A preliminary examination of the case with jail term and rehabilitation indicates faster dynamics of the system. For certain parameters, the number of drug users dies of relatively quickly. This tells us that a combination of strict jail sentencing combined with an effort to remove people from the drug-abusing class is an effective way of combating crack-cocaine abuse, but further investigation is necessary before any strong conclusions can be drawn from this model.
7 Conclusions

The war of drugs has been a long drawn out affair and we are losing. The victims of this drug war are all who are addicted, regardless of their age, race or gender. Drug addiction is not an easy process to endure. Children are our most precious assets and it is essential that we protect them.

Our first model is very simple to analyze and provides a nice starting point for our investigations. An endemic solution exists and $\beta_f$ is the most important parameter in determining the level of crack-cocaine abuse in the population of pregnant and non-pregnant women. The driving force behind our system is the non-linear $\frac{D_m N_m}{N}$ term, and $\beta_f$ is responsible for scaling how much influence that term has. We have also shown that $\gamma$ and $\rho$ are also important parameters. If we were to introduce an educational program, then we would want that program to alter these parameters. Which parameters we decided to alter depends on the intent of our educational program. If we want to reduce the total number of crack-cocaine using women, then we would want to focus on altering $\beta_f$. If we wanted to focus on the population of pregnant women in rehabilitation, we would focus on reducing $\rho_i$, the rate that women relapse back into drug use.

Including a drug-induced mortality rate is a very important alteration to our system. It can have a dramatic impact on the dynamics and make the model more realistic. Unfortunately the added complexity make the system more difficult to analyze and we were unable to analytically find the stability of an endemic solution. From this model we were able to conclude that it was feasible to reduce the amount of crack-cocaine abuse to very low levels, although not necessarily to eliminate it completely. This is encouraging in that it indicates that it may be able to reduce if not eliminate the prevalence of drug abuse in real life.

The jail system has slower dynamics with results that do not show steady state behavior until after over 100 years in some cases. Not only is increasing the jail sentence for drugs not as effective as having a rehabilitation program, but it is also more expensive. This seems to indicate that going to jail is not enough to reduce the amount of drug abuse in our society. It is vital to have a rehabilitation system set up to keep people off of drugs. According to our model with a jail term, it would take an average sentence for drug abuse of 200 years before the prevalence of crack-cocaine would die out. Of course this is unrealistic, but that is the extreme to which we would have to go to in order to eliminate drug abuse from our society.

Now we need to consider which is more cost-effective, putting men in jail
or supporting drug rehabilitation programs. With the parameters we have
gotten from different governmental sources, we know that putting men in
jail is not as effective as a rehabilitation program. Furthermore, the cost of
keeping drug offenders in jail has almost tripled in five years from $8 billion
in 1993 to $21 billion in 1998 [21]; The cost of rehabilitation programs is
$4.4 billion a year [22]. Clearly it is more cost effective to concentrate on
public programs that focus on the rehabilitation of people who abuse crack-
ocaine than to try and incarcerate these drug-offenders. Not only would it
be necessary to increase the jail sentence to these individuals, but it would
perpetuate the problem of a skyrocketing corrections facilities costs. On the
other hand, rehabilitation programs are far more effective at keeping men,
women, and especially pregnant women off of drugs. It is evident that we
should shift our allocation of resources and spend greater efforts at increasing
the scope of various drug rehabilitation and education programs. Our money
would be much better spent if we tried to help people, not lock them behind
bars.

8 Future Work

An age structured model and stability analysis of the drug abuse free equilib-
rium will be left for future study. Stability analysis for the endemic equilibria
of the drug induced mortality models will also be left for future analysis. We
did not have time to fully investigate the dynamics of our model with a jail
term, or the model with a jail and rehabilitation term. Further investiga-
tion of these models and simulations could provide potentially interesting
results regarding periodic states that may or may not be relevant to real life
applications.

9 Acknowledgments

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Appendix

Figure 6: Upper left: Increasing $\beta_m$, Upper right: Decreasing $\beta_m$, Lower left: Increasing $\beta_m$, Lower right: Decreasing $\beta_m$.

Figure 7: Upper left: Increasing $\gamma_i$, Upper right: Decreasing $\gamma_i$, Lower left: Increasing $\gamma_i$, Lower right: Decreasing $\gamma_i$. 
Figure 8:

Figure 9:
Figure 10:

Drug Using Women: $\beta_1 = 0.0714$, $\nu_1 = 0.22$, $\nu_2 = 0.01$, $R_0 = 0.48887$

Non-Pregnant Woman $D_n$

Pregnant Woman $P_n$

Recovered Women: $\gamma_{1}= 0.007106$, $\gamma_{2}= 0.014587$, $R_0 = 0.48887$

Non-Pregnant Woman $R_n$

Pregnant Woman $P_r$

Figure 11:

Drug Using Men in Jail: $\gamma_m = 0.06$, $\nu_m = 0.1$, $\beta_m = 0.00714$, $R_0 = 0.48887$

Drug Using Man $D_m$

Men in Jail $J_m$

24
Figure 12:

Figure 13:
Figure 14: Female Population

Figure 15: Male Population
Figure 16: Male Population

Figure 17: Female Population
Figure 18: Male Population

Figure 19: Changing $\beta_f$ can have major impacts on the population of drug using women. DUW = drug using women including pregnant women. DFW = Drug free women including pregnant women and women in rehabilitation. $\beta$ is $\beta_f$. 
Figure 20: The effects of $\beta_f$ on the population of pregnant women. $P_s$ is pregnant susceptible women, $P_d$ is pregnant crack-cocaine abusing women, $P_r$ is pregnant women in a rehabilitation program.

Figure 21: The effects of $\beta_m$ on the male population. DUM are crack-cocaine abusing men. DFM are men who do not abuse crack-cocaine. $\beta$ is $\beta_m$. 
Figure 22: The effects of $\gamma$ on pregnant women.

Figure 23: The effects of $\gamma$ on the total female population.
Figure 24: The effects of $\rho$ on pregnant women.

Figure 25: The effects of $\rho$ on the total female population.